EURYDICE

Mathematics Education in Europe:

Common Challenges and National Policies

European Commission

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FOREWORD

Competence in mathematics has been identified at EU level as one of the key competences for personal fulfilment, active citizenship, social inclusion and employability in the knowledge society of the 21st century. Concerns about low student performance, as revealed by international surveys, led to the adoption in 2009 of an EU-wide benchmark in basic skills which states that *'by 2020 the share of 15-year-olds with insufficient abilities in reading, mathematics and science should be less than 15 %'* ([1]). In order to achieve the target by 2020, we must identify obstacles and problem areas on the one hand and effective approaches on the other. This report, which is a comparative analysis of approaches to mathematics teaching in Europe, aims to contribute to a better understanding of these factors.

The report reviews national policies for reforming mathematics curricula, promoting innovative teaching methods and assessment, and improving teacher education and training. It calls for overarching policies for mathematics education that are based on continuous monitoring, research evidence. It also argues for comprehensive support policies for teachers, a renewed focus on the various applications of mathematical knowledge and problem-solving skills, and for the implementation of a range of strategies to significantly reduce low achievement.

The report also delivers recommendations on how to increase motivation to learn mathematics and encourage the take-up of mathematics-related careers. Many European countries are confronted with declining numbers of students of mathematics, science and technology, and face a poor gender balance in these disciplines. We need to urgently address this issue as shortages of specialists in mathematics and related fields can affect the competitiveness of our economies and our efforts to overcome the financial and economic crisis.

I am confident that this report, which is based on the latest research and extensive country evidence, will make a timely contribution to the debate on effective mathematics education. It will be of great help to all those concerned with raising the level of mathematical competence of young people in Europe.

Androulla Vassiliou
Commissioner responsible for
Education, Culture, Multilingualism and Youth

([1]) Strategic Framework for European Cooperation in Education and Training ('ET 2020'), Council Conclusions May 2008, OJL 119, 28.5.2009.

CONTENTS

INTRODUCTION

In recent years, the issue of competence in mathematics has become increasingly important and has been taken up at the highest policy level. Mathematical competence has been identified as one of the key competences necessary for personal fulfilment, active citizenship, social inclusion and employability in a knowledge society ([1]). Moreover, the 2008 *'Council Conclusions on preparing young people for the 21st century: an agenda for European cooperation on schools'* ([2]) considers the acquisition of literacy and numeracy skills to be the main priority for European cooperation in education.

> Numeracy, mathematical and digital competences and an understanding of science are also vital for full participation in the knowledge society and for the competitiveness of modern economies. Children's first experiences are crucial, but students are too often anxious about maths and some distort their learning choices to avoid it. Different teaching approaches can improve attitudes, raise attainment levels, and open up new learning possibilities. [COM (2008) 425 final]

The concerns about achievement levels have led to the establishment of an EU-wide benchmark in basic skills, to be achieved by 2020:

> ***'The share of 15-years olds with insufficient abilities in reading, mathematics and science should be less than 15 %'*** ([3]).

This benchmark is linked to one of the four strategic priorities for cooperation in education and training at EU level, i.e. improving the quality and efficiency of education and training. It is a means for monitoring progress and identifying challenges, as well as contributing to evidence-based policy making.

Objectives of the report

In the light of these policy developments, this first Eurydice report on mathematics education aims to contribute to European and national debate on how to improve the teaching and learning of mathematics and provide support to European cooperation in the field.

A range of factors influence the way mathematics is taught and learned. Results from international surveys suggest that education outcomes are related not only to students' family background, but also to the quality of teaching and to certain structural and organisation features of education systems. This study therefore examines the context in which mathematics learning takes place, the national policies which influence the teaching and learning of this crucial subject, and recent evidence from international surveys and research. It focuses on the instruments used by public authorities to improve mathematics education, including curricula, teaching methods, assessment arrangements, teacher education and support structures.

The report highlights the common challenges facing European countries and the national responses to these challenges. It reviews national policies for raising attainment levels, increasing motivation and overcoming barriers to learning in the light of evidence on what constitutes effective mathematics teaching. In doing so, the report identifies successful practices implemented in different education systems and suggests ways to tackle the issue of low achievement.

([1]) OJL 394, 30.12.2006.

([2]) 2008/C 319/08.

([3]) Strategic Framework for European Cooperation in Education and Training ('ET 2020'), Council Conclusions May 2008, OJL 119, 28.5.2009.

For the purposes of this study, mathematical competence will be understood to go beyond basic numeracy to cover a combination of knowledge, skills and attitudes. Mathematical competence will refer to the ability to reason mathematically, to pose and solve mathematical questions, and to apply mathematical thinking to solve real life problems. It will be linked to skills like logical and spatial thinking, the use of models, graphs and charts and understanding the role of mathematics in society. This approach is in line with the definitions used by the Council of the EU and the OECD ([4]).

Scope

The report provides information about 31 Eurydice network countries (the EU Member States, Iceland, Liechtenstein, Norway, and Turkey). The report covers ISCED levels 1 and 2 (primary and lower secondary education). References to ISCED level 3 (upper secondary education) have been made where appropriate. The year of reference is school year 2010/11.

Mathematics education in the public education sector only is examined, except in the case of Belgium, Ireland and the Netherlands, where the grant-aided private sector is also covered since it accounts for the majority of school enrolments. Moreover, in Ireland the vast majority of schools are legally defined as privately owned but, in fact, are fully state-funded and do not require the payment of fees by parents. In the Netherlands, equal funding and treatment of private and public education is enshrined in the constitution.

Report structure

The report begins with an overview chapter on *Achievement in mathematics: Evidence from international surveys* which discusses the major trends in achievement, as revealed by recent PISA and TIMSS surveys. It describes the conceptual framework of the international surveys, their main objectives and target populations and highlights some limitations in using and interpreting international survey results.

Chapter 1 *The Mathematics curriculum* presents an overview of the structure and content of the different steering documents (including the curriculum, syllabuses and official guidelines) for mathematics teaching. It examines the involvement of central education authorities in the production, approval and review of these documents. In addition, the recommended taught time for mathematics and national policies on the use of learning materials and textbooks are reviewed. Some information on the time allocated to various mathematical topics in the classroom is presented based on international survey results. Examples of national approaches to the production of textbooks and national strategies for assuring consistency between the curriculum and the learning materials used in the classroom are also described.

Chapter 2 *Teaching approaches, methods and classroom organisation* reviews research and policy developments in these areas. The analysis focuses on several teaching approaches and methods that are prescribed, recommended or supported in different European countries. These include problem-based learning, relating mathematics learning to daily life, active learning, critical thinking, the use of ICT, assigning homework and the grouping of pupils. This information is considered in the context of findings from international surveys which provide data on practices in schools. Finally, a discussion on the use of national surveys and reports for evidence-based policies on mathematics education is also provided.

([4]) Recommendation of the European Parliament and of the Council, of 18 December 2006, on Key competences for lifelong learning, Official Journal L 394 of 30.12.2006; The PISA 2003 Assessment Framework Mathematics, Reading, Science and Problem Solving Knowledge and Skills, OECD, Paris, 2003.

Chapter 3 *Assessment in mathematics* analyses central guidelines as well as practices related to different forms of assessment used for summative and formative purposes. The chapter looks also at national testing in mathematics and whether mathematics is included in school leaving examinations at the end of upper secondary education. The use of mathematics assessment data for improving the quality of teaching and supporting new policy developments is also briefly discussed.

Chapter 4 *Addressing low achievement in mathematics* presents an overview of research results on effective measures to improve achievement and outlines the main elements of national policies in this area. In addition, it reviews the tools used at national level to formulate evidence-based policies on low achievement. Lastly, it examines the use of specific forms of support including curriculum modification, diagnostic tools, one-to-one and small group tuition, and the intervention of specialised teachers.

Chapter 5 *Improving student motivation* provides an overview of the policies and initiatives to increase student motivation in learning mathematics. It presents national strategies and practices for fostering positive attitudes towards MST-related subjects. The chapter also highlights policy concerns related to the take-up of mathematics in higher education and skills shortages in the labour market. The issue of gender differences is addressed throughout the chapter not only because it has been the focus of attention in the research field but also because of the importance of policy measures to address girls' motivation to learn mathematics and improve their participation in higher education.

Chapter 6 *Education and professional development of mathematics teachers* highlights some of the key aspects of mathematics teacher education and professional development that enable teachers to provide students with high-quality learning opportunities. It starts with a profile of the mathematics teaching profession, followed by an analysis of existing policies and practices in European countries regarding initial teacher education and continuing professional development. These are presented against the background of the academic research literature in the field as well as data from the TIMSS and PISA international surveys and Eurydice's own survey on initial teacher education programmes in mathematics and science (SITEP).

The report also contains annexes on mathematics curriculum content and centrally promoted initiatives for teacher collaboration.

The comparative analysis is largely based on national responses to a questionnaire developed by the Eurydice Unit within the Education, Audiovisual and Culture Executive Agency. Data from TIMSS and PISA international surveys, as well as Eurydice's SITEP survey have also been extensively analysed.

The report has been checked by all Eurydice National Units and all contributors are acknowledged in a separate final section.

EXECUTIVE SUMMARY

The mathematics curriculum

The objectives, content and expected learning outcomes of mathematics education are generally defined in the curriculum. In recent years, the majority of countries have revised their mathematics curricula to bring into effect a stronger focus on competences and skills, an increase in cross-curricular links and a greater emphasis on the application of mathematics in everyday life. This learning outcomes-based approach tends to be more comprehensive and flexible in responding to the needs of learners.

However, the effective translation of curriculum objectives into classroom practice depends, amongst other things, on providing specific support and guidance to teachers and schools to deliver the new curricula.

Teaching approaches and methods

Research evidence suggests that effective mathematics instruction involves the use of a variety of teaching methods. At the same time, there is general agreement that certain methods such as problem-based learning, investigation and contextualisation are particularly effective for raising achievement and improving students' attitudes toward mathematics. While most central authorities in Europe report providing some form of national guidance on teaching approaches in mathematics, there is further potential for strengthening support for methods which promote students' active learning and critical thinking.

National guidelines on the use of calculators are rare, as is advice on homework and student grouping in mathematics. The use of ICT, on the other hand, is supported in all countries; however, international survey data show that ICT is not frequently used in mathematics lessons. More research and evidence about the benefits of ICT for mathematics instruction could help to promote and guide its effective use.

Assessment in mathematics

Student assessment in mathematics is a crucial element of the teaching and learning process and teachers play a key role in this. National guidelines for classroom assessment, particularly for innovative forms such as project-, portfolio-, ICT- or self/peer-based assessment exist only in a few countries. Mathematics teaching could benefit from increased support for schools and teachers on how to prepare and carry out assessment, and, most importantly, on how to provide relevant feedback to students.

National tests in mathematics are widely implemented and used to inform curriculum development as well as to improve teacher training and professional development. However, evidence in this report suggests that results could be used more systematically for policy formulation at all levels of decision-making.

Addressing low achievement

Research findings indicate that effective measures to tackle low achievement must be comprehensive, addressing a range of factors in and out of school, and they must also be timely. The majority of countries provide some national guidelines to address student difficulties in mathematics. However, effective guidance for schools and teachers and systematic support for students may require more targeted programmes, including the use of specialised teachers.

In order to effectively address low achievement in mathematics, student achievement needs to be monitored and progress must be measured. Currently, only a minority of countries have set national objectives for reducing low achievement. Research into the causes of low achievement in mathematics and the evaluation of support programmes are also rare, but indispensible for improving student outcomes.

Improving student motivation

The level of motivation to learn mathematics is an important determinant of students' achievement in school. National strategies for increasing student motivation exist in nearly half of the European countries examined. Most of these involve projects focusing, for example, on extra-curricular activities or partnerships with universities and companies, but large-scale initiatives covering all education levels and including a wide range of actions exist only in Austria and Finland. Targeted measures for students with low motivation and achievement which take into account the gender dimension also need to be scaled-up.

Motivation plays an important role in students' choice of further academic study and future career. Across Europe, the share of MST graduates compared to all other university graduates has been decreasing, and there has been no improvement in the share of female graduates in recent years. Many European countries have expressed concerns regarding these trends. In order to address them, existing actions need to be reinforced, in particular national campaigns and initiatives to attract more women into mathematics-related fields of study and professions.

Education and professional development of mathematics teachers

To be effective, mathematics teachers need a sound knowledge of the subject and a good understanding of how to teach it. In most European countries, initial teacher education programmes cover a wide range of areas of mathematical knowledge and teaching skills. This is echoed by the findings of the EACEA/Eurydice pilot survey of initial teacher education programmes (SITEP). However, both SITEP and official regulations and recommendations indicate that teaching mathematics to a diverse range of students and in a gender sensitive way are competences that need to be strengthened in future programmes for general as well as specialist teachers.

Most European countries promote teacher cooperation and collaboration in mathematics, mainly through interactive websites, in order to facilitate the exchange of information and experiences. A range of teaching approaches and methods are also covered in centrally promoted programmes for professional development. However, international survey results show that low participation rates in such programmes pose a problem that needs to be addressed. Incentives to promote participation in mathematics-related professional training are currently only offered in a small minority of European countries.

Promoting evidence-based policies

Raising the quality of mathematics teaching also depends on the collection, analysis and dissemination of evidence about effective practices. Currently, investigations on the use of teaching methods and assessment instruments are not widespread across Europe. Only a few countries have national structures in place for systematically gathering and analysing data on the development of mathematics teaching. The use of research evidence, evaluations and impact results to inform new policy decision needs to be strengthened. The aim of reaching the European objectives for reducing the level of low-achieving students in mathematics and increasing the number of graduates in mathematics-related fields should be supported by further monitoring and reporting at both national and European levels.

ACHIEVEMENT IN MATHEMATICS:
EVIDENCE FROM INTERNATIONAL SURVEYS

International student assessment surveys are carried out under agreed conceptual and methodological frameworks with a view to providing policy-oriented indicators. The relative standing of countries' average test scores is the indicator that attracts the most public attention. Since the 1960s, a country's relative score has become an important influence on national education policies, generating pressure to borrow educational practices from top-performing countries (Steiner-Khamsi, 2003; Takayama, 2008). This chapter presents the average test scores and standard deviations in mathematics achievement for European countries as reported by recent major international surveys. In addition, since the European Union member states have a political commitment to reduce the proportions of low achievers, the proportion of pupils lacking basic skills in mathematics is reported for each European country. Finally, basic information on the methodology of international surveys on mathematics achievement is also provided.

Cross-national research may help to explain the evident differences between and within countries as well as identify any specific problems present in education systems. However, the indicators from international surveys should be used cautiously as there are many important factors outside the realm of education policy which influence educational achievement and these often differ between countries. The country level indicators have been criticised as presenting simplified indicators of the performance of an entire school system (Baker and LeTendre, 2005). When interpreting the results it is also important to keep in mind that large-scale comparative studies face several methodological challenges: translations may generate different meanings; perceptions of some questions might be influenced by cultural bias; social desirability and pupil motivation may vary in different cultural contexts; even the political agenda of the organisations that conduct international assessments may influence assessment content (Hopmann, Brinek and Retzl, 2007; Goldstein, 2008). However, a number of quality control procedures are implemented to minimize the impact of these methodological problems on the comparability of results.

Major surveys on mathematics: TIMSS and PISA

Currently, student achievement in mathematics is assessed through two large scale international surveys, namely TIMSS and PISA. The Trends in International Mathematics and Science Study (TIMSS) provides data on the mathematics achievement of fourth and eighth grade students in various countries ([1]).

PISA (Programme for International Student Assessment) measures the knowledge and skills of 15-year-old students in reading, mathematics and science. Each PISA assessment cycle has a particular focus on one subject area. When mathematics was the main focus in 2003, it included questions relating to students' attitudes towards mathematics teaching. Trends in mathematics can only be calculated from 2003 (when mathematics was the major domain) to 2009 (the most recent results).

These two surveys focus on different aspects of student learning. In general terms, TIMSS aims to assess *'what students know'*, while PISA seeks to find *'what students can do with their knowledge'*. TIMSS uses the curriculum as the major organizing concept. The data collected has three aspects, the *intended curriculum* as defined by countries or education systems, the *implemented curriculum* actually taught by teachers, and the *achieved curriculum* or what students have learned (Mullis, Martin and Foy 2008, p. 25). PISA is not directly focused on any particular aspect of the curriculum, rather it

([1]) A few countries also conduct the so-called TIMSS 'advanced', which assesses students in their last year of secondary school who have studied advanced mathematics or physics aims to assess how well 15-year-old students can apply their knowledge of mathematics in everyday life. It focuses on mathematical literacy, which is defined as

> An individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments, and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen (OECD 2003, p. 24).

TIMSS is conducted every four years and the last round, which was conducted in 2007, is the fourth cycle of international mathematics and science assessments ([2]). Since fourth grade students subsequently become eighth grade students in the next cycle of TIMSS, those countries that participate in consecutive cycles also acquire information about relative progress across grades ([3]). However, only a few European countries have participated in all TIMSS surveys (namely Italy, Hungary, Slovenia and the United Kingdom (England)). Generally, less than half of the EU-27 countries participate in TIMSS. In the last round of the survey, 15 Eurydice network education systems measured mathematics and science achievement at the fourth grade and 14 measured achievement at the eighth grade.

PISA, on the other hand, covers almost all European education systems. The latest round of PISA (2009) involved the majority of European countries, including all education systems in the Eurydice Network with the exceptions of Cyprus and Malta. PISA 2003, which focused on mathematics, was not carried out in either of these two countries nor in Bulgaria, Estonia, Lithuania, Romania and Slovenia.

TIMSS uses grade-based samples and PISA uses age-based samples. The differences in assessed student population yield certain consequences. In TIMSS all students have received a similar amount of schooling, e.g. they are in the fourth or eighth school year ([4]), but their ages differ across participating countries depending on school starting ages and grade retention practices (see more in EACEA/Eurydice (2011)). For example, in TIMSS 2007, the average age of fourth graders in European countries at the time of testing was between 9.8 and 11.0 (Mullis, Martin and Foy 2008, p. 34) and the age of eighth graders between 13.7 to 15.0 (Ibid., p. 35). In PISA all respondents are 15 years old, but the number of completed school years differs, especially in those countries where grade retention is practiced. The average grade of the 15-year-olds tested in 2009 across all European countries varied from 9 to 11, but in some countries students who completed the test came from six different grades (from 7th to 12th).

As TIMSS is focused on the curriculum, it gathers a broader array of background information relating to student learning environments than PISA. Sampling entire classes within schools enables information to be gathered from the teachers teaching mathematics to those classes. Teachers complete questionnaires about the teaching methods used to implement the curriculum, their initial teacher education and their continuing professional development. In addition, the school heads of the assessed students provide information about school resources and the school climate for learning. Students are asked about their attitudes towards mathematics, school, interests and computer use. They also provide information about their home and classroom experiences.

([2]) For a description of the instrument development, data-collection procedures, and analytic methods used in TIMSS 2007 see Olson, Martin and Mullis (2008).

([3]) Due to the sampling methods used, the populations are not entirely the same, but are designed to be nationally representative.

([4]) The United Kingdom (England and Scotland) tested students in their fifth and ninth year of schooling, because their students start school at a very early age and otherwise would have been very young. Slovenia has been undergoing structural reforms requiring students to start school at a younger age so that students at the fourth and eighth grades would be the same age as students previously were in the third and seventh grades, but having had an additional year of schooling. To monitor this change, Slovenia assessed students in the third and seventh years of schooling in previous assessments. The transition has been completed at the fourth grade, but not at the eighth grade where some of the students assessed.were in the seventh year of schooling (Mullis, Martin and Foy 2008).

With respect to the learning context, PISA 2003 asked school heads to provide data about the school and the organisation of mathematics teaching. In addition to questions on their background and attitudes towards mathematics, students in 19 European countries completed an optional PISA questionnaire providing information about access to computers, how often they used them and for what purposes.

The TIMSS 2007 mathematics assessment framework was based on two dimensions: the content dimension and the cognitive dimension. At the fourth grade, the three areas of content were number, geometric shapes and measures, and data display. At the eighth grade, the four areas of content were number, algebra, geometry, and data and chance. The same cognitive dimensions – knowing, applying, and reasoning – were assessed in both grades (Mullis, Martin and Foy 2008, p. 24).

Mathematical literacy in PISA is assessed in relation to the four mathematical content areas: quantity; space and shape; change and relationships; and uncertainty. Questions were organised in terms of 'competency clusters' or skills needed for mathematics, namely reproduction (simple mathematical operations); connections (bringing together ideas to solve straightforward problems); and reflection (wider mathematical thinking).

In conclusion, TIMSS and PISA assessments were designed to serve a different purpose and are based on a separate and unique framework and set of questions. Thus differences between the studies in the results for a given year or trend estimates should be expected.

Mathematics achievement according to PISA findings

Results from PISA are reported using scales with an average score of 500 and a standard deviation of 100 set for students from all OECD participating countries. In 2003, when the standards for mathematics achievement were set, it could be inferred that approximately two-thirds of students across OECD countries scored between 400 and 600 points. The PISA mathematics scale is also divided into proficiency levels which differentiate and describe what a student can typically be expected to achieve by associating the tasks with levels of difficulty. Six proficiency levels were defined on the mathematics scale in 2003 and were used in the reporting of mathematics results for PISA 2006 and 2009 (OECD, 2009).

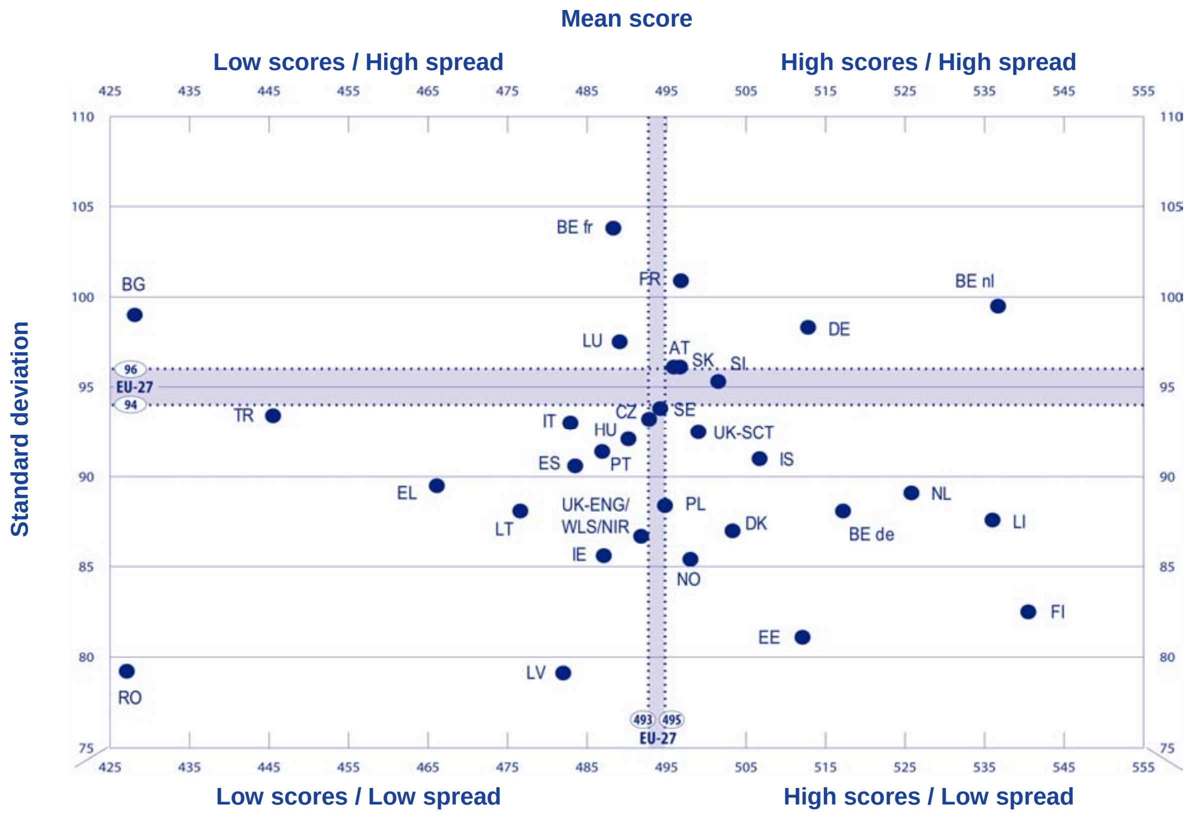
Average achievement is the most common indicator when comparing the performance of education systems in international student assessment surveys. In the EU-27 in 2009, the average mathematics performance was 493.9 ([5]) (see Figure 1.1). Finland had the highest results (540.1), but the Flemish Community of Belgium (536.7) and Liechtenstein (536) performed at similar levels (there were no statistically significant differences between the scores of these education systems). However, the best performing European education systems did less well than students in the top performing countries/regions worldwide (Shanghai-China (600), Singapore (562), and Hong-Kong-China (555)), but performed at about the same level as students in Korea (546) and Chinese Taipei (543).

At the other end of the scale, students in Bulgaria, Romania and Turkey had a considerably lower average achievement than their counterparts in all other participating Eurydice countries. The mean scores in these countries were between 50 and 70 points lower than the EU-27 average.

([5]) This is an average estimate taking into account the absolute size of the sampled population of 15-year-olds in each EU-27 country that participated in PISA 2009. The EU-27 average score was constructed in the same way as the OECD total (i.e., the average across OECD countries, taking absolute sample size into account). The OECD total in 2009 was 488 (OECD, 2010a).

Only 11 % of the variation in student performance lies between countries ([6]). The remaining variation lies within countries, i.e. between education programmes, between schools, and between students within schools. The relative distribution of the scores within a country, or the gap between the highest and the lowest achieving students, serves as an indicator of equity in educational outcomes. In the EU-27 in 2009, the standard deviation in mathematics achievement was 95.0 (see Figure 1), which means that approximately two-thirds of students in EU-27 scored between 399 and 589 points.

Figure 1: Mean score and standard deviation in mathematics for 15-year-old students, 2009



	EU-27	BE fr	BE de	BE nl	BG	CZ	DK	DE	EE	IE	EL	ES	FR	IT	CY	LV	LT	LU
Mean score 2009	494	488	**517**	**537**	**428**	493	**503**	**513**	**512**	**487**	**466**	**484**	497	**483**	x	**482**	**477**	**489**
Difference from 2003	**-5.2**	-9.3	2.1	**-16.7**	x	**-23.7**	**-11.0**	**9.8**	x	**-15.7**	**21.2**	-1.6	**-14.0**	**17.2**	x	-1.4	x	-4.1
Standard deviation 2009	95	**103.8**	**88.1**	**99.5**	99	93.2	**87**	**98.3**	**81.1**	**85.6**	**89.5**	**90.6**	**100.9**	93	x	**79.1**	**88.1**	**97.5**
Difference from 2003	-1.3	-4.0	**-12.2**	**-5.8**		-2.7	**-4.3**	-4.3		0.3	-4.3	2.1	**9.2**	-2.7	x	**-8.8**		**5.6**

	HU	MT	NL	AT	PL	PT	RO	SI	SK	FI	SE	UK (1)	UK-SCT		IS	LI	NO	TR
Mean score 2009	490	x	**526**	496	495	**487**	**427**	**502**	497	**541**	494	492	499		**507**	**536**	498	**446**
Difference from 2003	0.2	x	**-12**	m	4.6	**20.9**	x	x	-1.5	-3.8	**-14.8**	m	**-24.8**		**-8.4**	0.2	2.8	**22.1**
Standard deviation 2009	92.1	x	**89.1**	96.1	**88.4**	**91.4**	**79.2**	95.3	96.1	**82.5**	93.8	**86.7**	92.5		**91**	87.6	**85.4**	93.4
Difference from 2003	-1.4	x	-3.4	m	-1.8	3.8			2.8	-1.2	-0.9	m	**8.2**		0.6	-11.5	**-6.6**	-11.3

m Not comparable x Countries not participating in the study

Source: OECD, PISA 2003 and 2009 databases.

UK (1) = UK-ENG/WLS/NIR

([6]) As computed by a 3-level (country, school and student) multilevel model for participating EU-27 countries.

Explanatory note

Two shadowed areas mark the EU-27 averages. These are interval indicators that take into account the standard errors. For readability, country averages are shown as dots but it is important to keep in mind that they are also interval indicators. The dots that approach the EU average area may not differ significantly from the EU mean. Values that are statistically significantly different (p<.05) from the EU-27 mean (or from zero when considering differences) are indicated in bold in the table.

Country specific notes

Austria: The trends are not strictly comparable as some Austrian schools boycotted PISA 2009 (see more OECD 2010c). However, Austrian results are included in the EU-27 average.

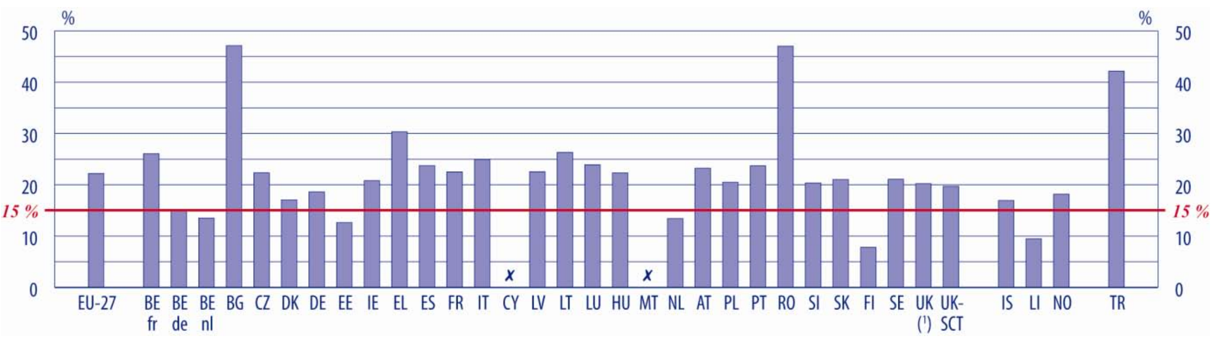
United Kingdom (ENG/WLS/NIR): PISA 2003 sample did not meet the PISA response-rate standards; therefore trend estimations are not possible. See OECD (2004, pp. 326-328).

Countries with a similar level of average performance can have different ranges of student scores. Therefore, when making comparisons between countries, it is important to consider not only a country's average student score but also its range of scores. Figure 1 unites these two indicators showing on the x axis countries' average results (proxy for efficiency of education systems) and on the y axis the standard deviation (proxy for equity of education systems). Countries that have a significantly higher average result and significantly lower standard deviations than the EU-27 can be considered both efficient and equitable in educational outcomes (see Figure 1, low right quarter). For mathematics achievement, Belgium (German-speaking Community), Denmark, Estonia, the Netherlands, Finland and Iceland could be considered as efficient and equitable education systems.

In Belgium (French and Flemish Communities), Germany, France and Luxembourg, the gap between high- and low-achieving students was especially large in 2009 (see Figure 1, top half). Schools and teachers in these countries therefore need to cope with a wide range of student abilities. Consequently, one way to improve a country's overall performance might be to concentrate on supporting low achievers (see more in Chapter 4).

Lastly, there are several European countries where the average performance in mathematics was lower than the EU average, although the spread of student achievement was not high. Ireland, Greece, Spain, Latvia, Lithuania, Portugal and Romania thus need to address mathematics performance across a range of proficiency levels in order to increase their average performance.

Figure 2: Percentage of low achieving 15-year-old students in mathematics, 2009



	EU-27	BE fr	BE de	BE nl	BG	CZ	DK	DE	EE	IE	EL	ES	FR	IT	LV	LT	LU
2009	22.2	26.1	15.2	13.5	47.1	22.3	17.1	18.6	12.6	20.8	30.3	23.7	22.5	24.9	22.6	26.3	23.9
Δ	1.3	2.9	-2.6	2.1	x	**5.8**	1.6	-3.0	x	**4.0**	**-8.6**	0.8	**5.9**	**-7.0**	-1.2	x	2.2
	HU	**NL**	**AT**	**PL**	**PT**	**RO**	**SI**	**SK**	**FI**	**SE**	**UK (¹)**	**UK-SCT**		**IS**	**LI**	**NO**	**TR**
2009	22.3	13.4	23.2	20.5	23.7	47.0	20.3	21.0	7.8	21.1	20.2	19.7		17.0	9.5	18.2	42.1
Δ	-0.7	2.5	m	-1.6	**-6.4**	x	x	1.1	1.1	**3.8**	m	**8.4**		2.0	-2.8	-2.7	**-10.1**

Δ – Difference from 2003   m – Not comparable   x – Countries that did not participate in the study

Source: OECD, PISA 2003 and 2009 databases.   UK (¹): UK-ENG/WLS/NIR

Explanatory note

Low achievers – defined as students who attain below Level 2 (<420.1).

When considering differences, values that are statistically significantly (p<.05) different from zero are indicated in bold.

Country specific notes

Austria: The trends are not strictly comparable as some Austrian schools have boycotted PISA 2009 (see more OECD 2010c). However, Austrian results are included in the EU-27 average.

United Kingdom (ENG/WLS/NIR): PISA 2003 sample did not meet the PISA response-rate standards; therefore trend estimations are not possible. See OECD (2004, pp. 326-328).

The proportion of students who do not have basic skills in mathematics is another important indicator of education quality and equity. The EU member states have set a benchmark to reduce the proportion of 15-year-olds with low achievement in mathematics to less than 15 % by 2020 ([7]). Students not reaching Level 2 in PISA are considered low achievers by the European Council. According to the OECD (2009), students reaching Level 1 have such a limited mathematical knowledge that it can only be applied to a few, familiar situations. They can carry out single-step processes that involve the recognition of familiar contexts and mathematically well-formulated problems, reproduce well-known mathematical facts or processes, and apply simple computational skills (OECD 2003, p. 54). Students performing below Level 1 are unable to demonstrate mathematical literacy in situations required by the easiest PISA tasks, which may hinder their participation in society and the economy.

As Figure 2 shows, in the EU-27 in 2009, an average 22.2 % of students were low achievers in mathematics. Only Estonia, Finland and Liechtenstein have already achieved the benchmark (i.e. the numbers of low achievers in mathematics are significantly lower than 15 %). In addition, the number of low achievers was approximately 15 % in Belgium (German-speaking and Flemish Communities) and the Netherlands. In contrast, the proportion of students lacking basic skills in mathematics was especially high in Bulgaria, Romania and Turkey – over 40 % of students in those countries did not reach Level 2.

Examining the EU-27 average trends in mathematics achievement since PISA 2003, there was a slight decrease (-5.2 points, standard error 2.34), but there was no statistically significant change in the standard deviation or the proportion of low achievers. However, methodologically, it is more appropriate to compare only those countries that participated in PISA 2003 and those that have comparable results in both assessments (i.e. excluding Bulgaria, Estonia, Lithuania, Austria, Romania, Slovenia and the United Kingdom (ENG/WLS/NIR)) ([8]). Comparing only these countries, the difference in average scores was not significant (-0.1 points, standard error of the difference 1.35) and there was no change in standard deviation (difference -1.4, standard error 0.84). The average proportion of students performing below Level 2 has also remained constant (difference -0.2 %, the standard error of the difference 0.55).

Several countries experienced considerable changes in mathematics performance between 2003 and 2009. Greece, Italy, Portugal and Turkey had significant improvements in their average score and reductions in the proportion of low achievers. In addition, in Germany, the average scores increased, but the proportion of students not reaching the proficiency Level 2 remained stable. Conversely, the decrease in average mathematics score was significant in the Flemish Community of Belgium, the Czech Republic, Denmark, Ireland, France, the Netherlands, Sweden and Iceland. The proportion of low achievers also increased in the Czech Republic (+5.8 %), Ireland (+4.0 %), France (+5.9 %) and Sweden (+3.8 %).

([7]) Council conclusions of 12 May 2009 on a strategic framework for European cooperation in education and training ('ET 2020'). OJ C 119, 28.5.2009.

([8]) For the methodological reasons for exclusion of comparisons see OECD (2010c, p. 26) and OECD (2004, pp. 326-328).

Mathematics achievement according to TIMSS findings

TIMSS scales were established using similar methodology as PISA. The TIMSS mathematics scales for the fourth and eighth grades are based on the 1995 assessments, setting the average of the mean scores of the countries which participated in TIMSS 1995 to 500 and the standard deviation to 100 (Mullis, Martin and Foy, 2008).

Due to the fact that relatively few European countries participate in TIMSS and not always the same countries test students in both fourth and eighth grades, this section will not draw heavily on comparisons with the EU average. Instead, the discussion will focus on differences between countries. The EU average ([9]) is provided in Figure 3 as an indication only.

Figure 3: Mean scores and standard deviations in mathematics achievement, students at fourth and eighth grades, 2007

Grade 4			Grade 8	
Mean score	Standard deviation		Mean score	Standard deviation
521.0	77.0	**EU-27**	492.8	84.9
x		**BG**	**463.6**	**101.6**
486.4	**71.5**	**CZ**	**503.8**	**73.7**
523.1	**70.8**	**DK**	x	
525.2	**68.2**	**DE**	x	
506.7	77.0	**IT**	**479.6**	**76.2**
x	x	**CY**	**465.5**	**89.3**
537.2	**71.9**	**LV**	x	
529.8	75.8	**LT**	**505.8**	**79.7**
509.7	**91.2**	**HU**	**516.9**	84.7
x	x	**MT**	487.8	**91.8**
535.0	**61.4**	**NL**	x	
505.4	**67.9**	**AT**	x	
x	x	**RO**	**461.3**	**99.8**
501.8	**71.4**	**SI**	**501.5**	**71.6**
496.0	84.9	**SK**	x	
502.6	**66.5**	**SE**	491.3	**70.1**
541.5	**86.0**	**UK-ENG**	**513.4**	83.6
494.4	78.9	**UK-SCT**	487.4	**79.7**
473.2	76.2	**NO**	**469.2**	**65.7**
x	x	**TR**	**431.8**	**108.7**

Country specific notes

Denmark and **United Kingdom (SCT)**: Met guidelines for sample participation rates only after replacement schools were included.

Latvia and **Lithuania**: National target population does not include all of the International Target Population as defined by TIMSS. Latvia included only students taught in Latvian, Lithuania only students taught in Lithuanian.

Netherlands: Nearly satisfied guidelines for sample participation rates after replacement schools were included.

United Kingdom (ENG): At eighth grade met guidelines for sample participation rates only after replacement schools were included

Values that are statistically significantly (p<.05) different from EU-27 mean are indicated in bold in the table.

Source: IEA, TIMSS 2007 database.

As Figure 3 shows, fourth grade students in Latvia (only students taught in Latvian), Lithuania (only students taught in Lithuanian), the Netherlands and the United Kingdom (England) had significantly higher achievement than the average in participating EU countries in 2007. The results were nevertheless significantly lower than the worldwide top performers (Hong Kong SAR (607 points), Singapore (599), Chinese Taipei (576) and Japan (568)) and were similar to Kazakhstan (549) and the Russian Federation (544).

([9]) This is an average estimate taking into account the absolute size of the population in each EU-27 country participating in TIMSS 2007.

At the eighth grade, the top performing European education systems were the Czech Republic, Hungary, Lithuania, Slovenia and the United Kingdom (England). Their results ranged between 500 and 520. However, they were significantly below the top performers in the world (average results in Chinese Taipei, Republic of Korea, Singapore, Hong Kong SAR and Japan were between 570 and 600).

At the other end of the scale, at the fourth grade, Norway (473 points) had significantly lower average results than all other participating European countries. The results of the Czech Republic, Italy, Hungary, Austria, Slovenia, Slovakia, Sweden, the United Kingdom (Scotland) were also lower that the EU average. At the eighth grade, Turkish students had much lower results than all other European countries (432 points). The results were also significantly lower than the EU average in Bulgaria, Italy, Cyprus, Romania and Norway.

It is important to take into account that the results for the fourth and eighth grades are not directly comparable. Even though 'the scales for the two grades are expressed as the same numerical units, they are not directly comparable in terms of being able to say how much achievement or learning at one grade equals how much achievement or learning at the other grade' (Mullis, Martin and Foy 2008, p. 32). Still, comparisons can be made in terms of relative performance (higher or lower). Therefore, for those countries that tested at both grades, it can be concluded that Lithuania and the United Kingdom (England) maintained a high performance at fourth and eighth grades.

As discussed previously, it is important to consider not only the average results, but also their spread, or the difference between low and high achieving students. At the fourth grade, Hungary and the United Kingdom (England) had significantly higher standard deviations than other participating education systems. In general, the spread of student results was fairly low in all European countries, i.e. lower than the international standard deviation set at 100. The standard deviation in the Netherlands (62 points) was much lower than in all other European countries.

At the eighth grade, conversely, there were five countries (Bulgaria, Cyprus, Malta, Romania and Turkey) with much higher differences between the results of high and low achieving students than in other European countries. On the other hand, Norway with 65 points had the lowest standard deviation. Unfortunately, there were very few students achieving high results and very many achieving low in Norway ([10]).

From the first TIMSS assessment in 1995, many countries experienced considerable changes in their average scores. At the fourth grade, very high increases in scores were observed in Latvia (38 points), Slovenia (41 points) and the United Kingdom (England) (57 points). At the eighth grade, such robust increases were reported only in Lithuania (34 points), but there was also a significant improvement in the United Kingdom (England) (16 points). In some other countries the results deteriorated. The mathematics performance of students in the Czech Republic decreased significantly at both fourth (54 points) and eighth (42 points) grades. Strong negative trends were also observed at the eighth grade in Bulgaria (67 points), Sweden (48 points) and Norway (29 points).

([10]) 0 % of eighth grade students in Norway reaching the Advanced Benchmark (625 points) and 48 % reaching only the Low Benchmark (400 points) (see Mullis, Martin and Foy 2008, p. 71).

Main factors associated with mathematics performance

International student achievement surveys explore factors associated with science performance on several levels: characteristics of individual students and their families, teachers and schools, and education systems.

Impact of home environment and individual student characteristics

Research has clearly established that **home background** is very important for school achievement (for an overview, see Breen and Jonsson, 2005). According to PISA, home background, measured on an index summarising each student's economic, social and cultural status, remains one of the most powerful factors influencing performance. TIMSS also reports a strong relationship between pupils' mathematics achievement and student background, measured as the number of books at home or speaking the language of the test at home (Mullis, Martin and Foy, 2008). However, poor performance in school does not automatically follow from a disadvantaged home background.

Positive **attitudes to mathematics** and self-confidence in learning mathematics are associated with higher mathematics achievement. These motivational aspects influence decisions about participation in school tracks or study programmes where mathematics is an important subject. These attitudes may shape students' post-secondary education and career choices (more on student attitudes, motivation and achievement in mathematics see in Chapter 5).

Gender differences in mathematics are not straightforward. On average, boys and girls have similar results in mathematics in most countries throughout most of their school years. TIMSS shows no consistent gender gap among pupils at the fourth and eighth grades. PISA reported some male advantage in all rounds although not in all countries. Boys' advantage over girls is, however, noticeable where students are assigned to different tracks, streams or study programmes. Overall gender averages are influenced by male and female student distribution across these different streams or tracks and, in many countries, although more females attend schools and tracks with higher than average performance, within these schools and tracks, they tend to achieve worse results in mathematics than males (EACEA/Eurydice, 2010; OECD, 2004).

Moreover, PISA 2003 results showed marked differences between males and females in their interest in and enjoyment of mathematics, their self-related beliefs, as well as their emotions related to mathematics. On average, females tended to report lower levels of interest and enjoyment in mathematics. Girls also tended to have higher anxiety levels regarding mathematics. Conversely, on average, boys had higher self-efficacy, i.e. a higher level of confidence in tackling specific tasks. Boys also had higher levels of belief in their mathematical abilities than girls, i.e. self-concept (OECD, 2004).

Impact of schools and education systems

International student achievement surveys are often used for country comparison. Yet, according to PISA 2009, differences between European countries explain only 10.5 % of the total variance of mathematics performance, while between-school differences represent approximately 35.4 % and within schools approximately 54.1 % of total variance ([11]). The degree to which students' educational chances are affected by which country they live in therefore should not be exaggerated. Yet, it is possible to distinguish certain features of education systems that can be associated with general student achievement levels and/or proportions of low achievers.

For example, PISA found that in countries where more students repeat grades, overall results tend to be worse and social disparities tend to be larger. Also, in countries and schools where students are

([11]) The numbers are computed by a 3-level (country, school and student) multilevel model for participating EU-27 countries.

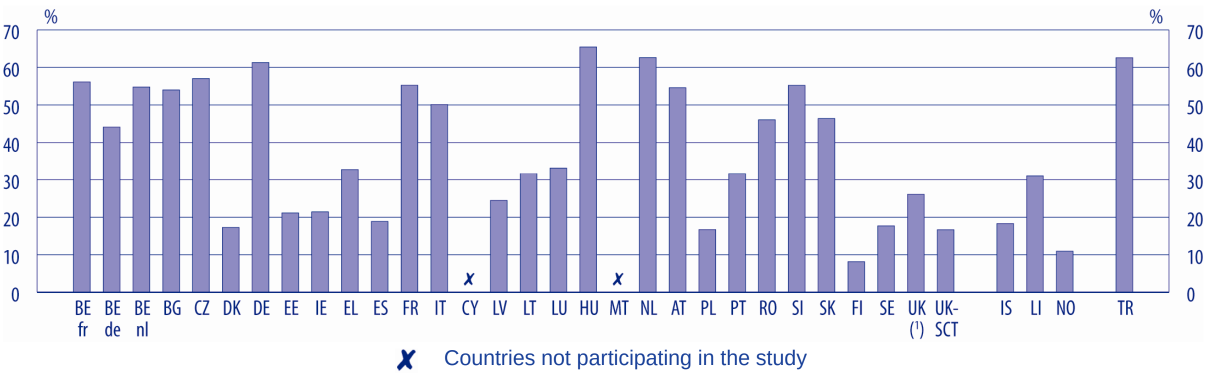
assigned to different tracks/streams based on their abilities, overall performance is not improved, but socio-economic differences are enhanced. In addition, in the education systems where selection takes place at a younger age larger social differences tend to be apparent (OECD 2004, pp. 263-264). These tendencies are consistent in every round of PISA assessment and are also valid for achievement in reading and science.

The school-level factors which contribute to higher student achievement vary to a great degree from country to country and their effects need to be interpreted by taking national cultures and education systems into account. The variation in student achievement that is observed within schools or between schools differs greatly across countries. Figure 4 shows a breakdown of variance in student mathematics performance in 2009. The length of the bars represents the percentage of the total differences in mathematics achievement that depends on school characteristics. In 12 educational systems most of the variation in student achievement was due to differences between schools. Between-school variation explained more than 60 % of differences in student achievement in Germany, Hungary, the Netherlands and Turkey. In these countries, schools, to a great extent, determined the learning outcomes of the student.

Usually, in those education systems where the number of school types or distinct educational programmes available for 15-year-olds is higher, the between-school variation is also higher (OECD 2004, p. 261). Other possible reasons for high differences between schools might be the varied socio-economic and cultural background of students entering the school, geographical disparities (such as between regions, provinces or states in federal systems, or between rural and urban areas), and the discrepancies in quality or effectiveness of mathematics teaching in various schools (OECD, 2004).

In contrast, in Finland and Norway only 8-11 % of the variation lay between schools. In these educational systems schools were rather similar.

Figure 4: Percentage of total variance explained by between-school variance on the mathematics scale for 15 year-old students, 2009



BE fr	BE de	BE nl	BG	CZ	DK	DE	EE	IE	EL	ES	FR	IT	LV	LT	LU
56.1	44.1	54.8	54.0	57.1	17.2	61.3	21.1	21.4	32.8	18.9	55.2	50.1	24.5	31.7	33.2
HU	**NL**	**AT**	**PL**	**PT**	**RO**	**SI**	**SK**	**FI**	**SE**	**UK (1)**	**UK-SCT**	**IS**	**LI**	**NO**	**TR**
65.5	62.6	54.6	16.7	31.6	46.1	55.2	46.4	8.2	17.7	26.1	16.6	18.3	31.0	10.9	62.6

Source: OECD, PISA 2009 database.

UK (1): UK-ENG/WLS/NIR

Both TIMSS and PISA conclude that in most countries the social background of a school (measured as the proportion of socially disadvantaged students or the average socio-economic status) is strongly associated with mathematics performance. The advantage resulting from attendance at a school where many students have favourable home backgrounds relates to a variety of factors, including peer-group influences, a positive climate for learning, teacher expectations, and differences in the resources or quality of schools. TIMSS results show that at both grades, on average, there was a positive association between attending schools with fewer students from economically disadvantaged homes and mathematics achievement. Also, achievement was highest among students attending schools with more than 90 percent of students having the language of the test as their native language (Mullis, Martin and Foy, 2008).

Moreover, PISA 2003 showed that a school's socio-economic context is much more predictive for mathematics achievement than individual students' socio-economic differences. This relation is often reinforced by the tracking or streaming of students into different study programmes. In countries with a larger number of distinct programme types, socio-economic background tends to have a significantly larger impact on student performance (OECD 2004, p. 261).

Explaining changes in mathematics achievement in some countries

Explaining changes in a particular country's results is rather difficult. The effects of any particular education reform are not immediate and significant trends are usually related to the combined influence of several factors. However, a number of analytical papers and reports can shed some light on the issue. A Swedish analysis (Skolverket, 2009) of declining student achievement highlights the influence of increasing segregation in the Swedish school system and the negative effects of decentralisation and streaming. Individualisation in teaching practices, or a shift of responsibility away from teachers to pupils, also had a negative impact. These factors increased the effect of student socio-economic background, whether by higher concentration of pupils from similar backgrounds in the same schools or strengthening the importance of home support, where parents' level of education assumed greater significance for pupils' educational attainments. Similarly, the curriculum for the 10-year compulsory school in Norway, introduced in 1997 (L97), emphasised that students should be independent, proactive and 'learn by doing'. Analysis of declining Norwegian results in the international surveys revealed that students were sometimes left alone to construct their knowledge from a multitude of experience (University of Oslo, 2006) and the teacher's role was being reduced to facilitator (Kjærnsli et al., 2004). TIMSS also contributed to a broad discussion about reasons for Norwegian students' especially low achievement on items requiring exact calculations compared to other countries. It was taken into account when revising the recent school curriculum in 2006/07, e.g. putting more focus on basic number skills.

There were also some positive examples. Reforms in Portugal (see more in Chapter 1) concentrated on improving learning opportunities for students and adults from disadvantaged backgrounds, including direct subsidies (in form of books, meals, laptops, etc.). In addition, grade repetition was reduced and a new system of evaluating teachers and schools was put in place together with emphasis on teacher training. These efforts were reinforced by the Action Plan for Mathematics (launched in 2005) (OECD, 2010c). Average achievement increased and the proportion of low achievers in mathematics decreased significantly in Portugal. Similar trends were observed in Turkey where the improvement in results may be related to the introduction of legislation on compulsory education and a drastic increase in participation in the eight-year education programme. This was supported by the introduction of new curricula, a revision of teacher education and the allocation of additional resources to school infrastructure, including libraries, ICT, reduced class sizes, etc. (Isiksal et al., 2007; OECD, 2010c).

General improvement or deterioration in mathematics achievement are usually associated with the teaching of all other basic skills, and are often linked to a general restructuring of the education system. In addition, changes in student performance can also signal changes in demographic conditions and in the socio-economic composition of student populations.

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The international student achievement surveys provide a wealth of information on mathematics achievement but they largely focus on individual and school factors; they do not systematically gather data on education systems (PISA) or analyse such data (TIMSS) with a view to assessing its impact on student mathematics achievement. This study examines the qualitative data on various aspects of European education systems with a view to identifying the main factors affecting mathematics performance and highlights good practices in the teaching of mathematics.

CHAPTER 1: THE MATHEMATICS CURRICULUM

Introduction

The learning aims, objectives and subject content of the mathematics curriculum are set down in various types of official documents including curriculum documents, guidelines for schools and teachers, and subject syllabuses (or school subject plans in some countries). These documents are referred to here as 'steering documents'. Different levels of government or school authorities may be involved in their development and approval, and information about them is disseminated in various ways.

All countries have a process of review of the steering documents which takes into account a range of evidence and opinion including the results of student assessment and the outcomes of school evaluation. This review process ensures that subject content, learning aims and outcomes keep pace with the challenges of modern society and the skills needed in the labour market. Moreover, the curriculum does not operate in isolation. Other factors such as the length of time students are taught mathematics (taught time), the organisation of teaching and the methods used, as well as the forms and criteria of assessment applied in primary and secondary education all make an important contribution to student achievement. The differences between countries in these areas, therefore, may go some way to explaining differences in levels of mathematics achievement across Europe.

This chapter presents an overview of the mathematics curriculum as presented in the various steering documents for the teaching of mathematics. It examines the involvement of different levels of education authority in the development and approval of these documents and considers arrangements for the monitoring and review of the curriculum. The learning objectives for mathematics as well as subject content and skills to be mastered are explored, and information (based on international survey results) is provided on the taught time actually spent on a range of mathematical topics. In addition, the recommended taught time for mathematics as a whole, and national policies on the use of learning materials and textbooks are examined. In the last section of the chapter we give some examples of national strategies for assuring consistency between the official curriculum and what is taught in schools through mathematics textbooks and other learning materials. More detailed information on specific teaching methods and the organization of mathematics education can be found in Chapter 2 'Teaching Approaches, Methods and Classroom Organisation '.

1.1. Development, approval and dissemination of mathematics steering documents

In the majority of European countries, the mathematics curriculum is established as a formal document that is often prescriptive. It specifies what topics must be learnt, describes the programmes of study and their content, as well as the teaching, learning, and assessment materials that should be used (Kelly, 2009). However, in some countries, no strict demarcations exist between the official curriculum document and other steering documents such as mathematics syllabuses or school subject plans. The latter are usually used for planning teaching courses and comprise information such as class time, an outline of course content, teaching methods or specific classroom rules (Nunan et. al 1988, p. 6). For this reason, when examining the decision-making authorities involved in the adoption or approval of mathematics steering documents, an open approach will be used towards both the presentation of current practices across Europe and their official status (e.g. compulsory or recommended). To simplify the analysis in the next sections all the documents produced at national level that include the general aims, outcomes and/or content of mathematics programmes will be treated as curriculum documents even when, in the national context, such documents are recognised as national syllabuses.

Levels of decision-making

In the vast majority of European countries, the curriculum is approved by central education authorities and is compulsory. It is usually set down in a central document which defines the objectives, learning outcomes and/or content of mathematics teaching.

In the **Czech Republic**, for example, 'framework educational programmes' (FEPs) are developed and adopted at central level. The FEP defines the binding framework for education at each stage of education (pre-primary, basic and secondary education). After the FEP is published, schools create 'school educational programmes' (SEP), which govern the teaching and learning in individual schools. SEPs are created by each school according to the principles set out in the relevant FEP. The degree of detail and the development of educational content for teaching mathematics is the responsibility of the school. Central authorities recommend the use of the *Manual for the development of school educational programmes*, which is created for each FEP ([1]) to guide the procedure for producing the different elements of the SEP and give concrete examples which can be followed by schools.

An equivalent process exists in **Slovenia** where binding documents at central level are defined as the 'basic school programme', which includes the 'basic school syllabus' and curricula for separate subjects that are commonly elements of the national curriculum. In accordance with the basic school programme, schools draft their annual work plan which specifies school activities, the scope and number of lessons and any extra-curricular activities. Mathematics teachers write their own annual plans in which they specify aims, standards of knowledge, and subject content.

In **Sweden,** a central level document with the characteristics of a national curriculum but entitled '*Syllabuses for compulsory education*' is developed centrally by the Swedish National Agency for Education. In each school and in each class, the teacher must interpret the national 'Syllabuses for compulsory education' that were implemented as from July 2011 and adapt the teaching process to pupils' abilities, experiences, interests and needs.

In **Norway** the national core curriculum and subject curricula must be interpreted and implemented locally. There is local autonomy regarding subject content and the teaching process.

In Belgium (French- and German-speaking Communities), the Netherlands, Romania and Slovakia schools are also involved in defining local curricula at different stages.

In **Belgium (French Community)**, the central level *'Core skills'* document *(Socles de Compétences)* (Decree of 26 April 1999) defines the minimum levels of competence for 8-, 12- and 14-year-old students. The various programmes adopted by the 'education networks' (school providers), must be in line with the *Socles de Compétences* and be approved by the Minister for Education. Each school belongs to a specific education network and must implement the education programmes in line with the *Socles de Compétences* and *Compétences terminales* (final skills) defined at higher level.

In the **Netherlands**, the attainment targets/learning outcomes are stipulated at central level and indicate what is known as 'competences' at the end of primary and secondary education. On this basis, the National Institute of Curriculum Development builds up a model/framework curriculum that schools can use in the development of their individual school plans. Schools have extensive autonomy in defining the subject content that will be taught to reach the attainment targets.

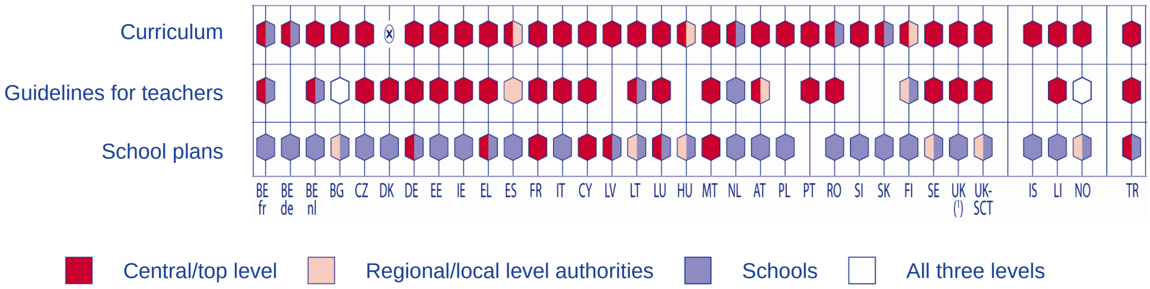
In Spain, Hungary and Finland, the mathematics curriculum is defined at two levels (central and regional/local) and school plans develop specific topics within this framework.

In **Finland**, the national core curriculum is designed by the National Board of Education (FNBE) and in **Hungary** the central level authorities adopt core curricula and also a set of recommended framework curricula. The second

([1]) http://www.vuppraha.cz/wp-content/uploads/2009/12/RVP_ZV_EN_final.pdf level of decision-making in both countries is the local level. The local curriculum is more detailed and integrates local elements but is developed in accordance with the National Core Curriculum. Finally, at school level, specific school plans which define detailed aims and content are developed and approved by teaching staff.

In **Spain**, the Ministry of Education establishes the national core curriculum at both primary and lower secondary education and, on the basis of these national core curricula, each Autonomous Community establishes its own curriculum. The national curricula do not include methodological guidelines for teachers, instead these are provided in the regional curricula of the Autonomous Communities. Moreover, in the Autonomous Communities there are also regulations relating to meeting the diverse needs of students. Finally, in addition to the regional curricula, schools have pedagogical autonomy to define and develop specific school plans according to their own socio-economic and cultural context.

Figure 1.1: Decision-making authorities involved in developing and approving the principal steering documents for mathematics teaching, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

UK (1) = UK-ENG/WLS/NIR

Country specific notes

Germany: The ministries of education in each *Land* are considered as the central/top level authority developing and approving the main steering documents for mathematics teaching.
Luxembourg: Syllabuses and school plans at primary level are developed by the Ministry of Education, and at lower secondary level mainly by schools.
Denmark: National authorities develop and publish a document entitled *Fælles Mål* which includes central guidelines and objectives for mathematics teaching, but this is not defined as a curriculum in national regulations.

In countries where guidelines for teachers exist, they are generally drafted at central level as recommendations and/or developed at school level. In countries where local authorities are responsible for education, they also may implement guidelines for teachers on the implementation of the mathematics curriculum.

In **Bulgaria,** all three decision-making levels take part in the development of documents which support teachers' work. The experts in the Ministry of Education, Youth and Science prepare a reference document related to the mathematics programme and learning content. In addition, the regional inspectorates of education develop teaching materials on specific topics. At school level, associations of mathematics teachers, including senior and head teachers, prepare guidance on the teaching methods appropriate for the mathematics programme.

In the vast majority of countries schools, either by themselves or with the support of the educational authorities, they create, approve and implement their own plans for mathematics teaching and establish their own rules for organising and running the institution. In general, schools have a large degree of autonomy in this area but they must usually take account of the centrally defined framework for mathematics.

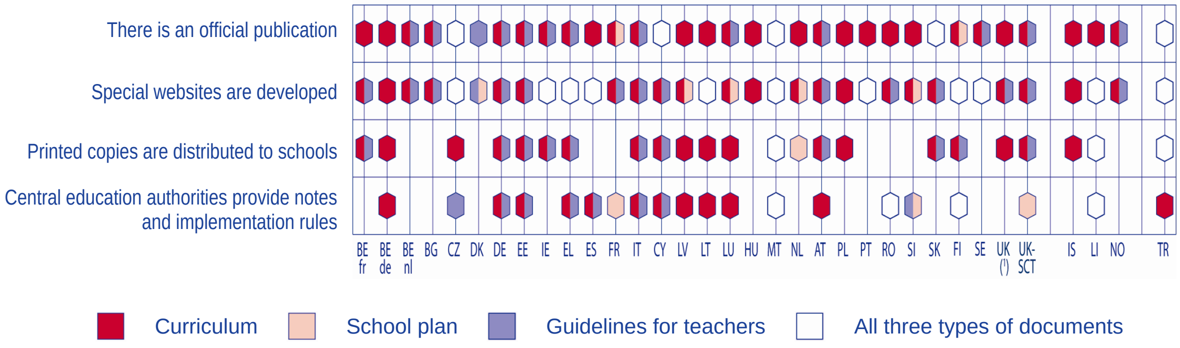
In **Bulgaria,** two different stages are in place: first teachers allocate the curriculum topics to particular lessons according to the defined taught time for a particular school year and, at a later stage, these allocations are approved by the school management for the compulsory part of the programme and by the regional inspectorates for the optional parts.

In **Latvia,** each school is obliged to have a syllabus for mathematics that is developed either by the school or selected from the example programmes developed by the State Education Centre ([2]).

Dissemination of information about curriculum changes

Change in education is a complex process and requires careful planning, sufficient time for implementation and adequate funding. The provision of support for teachers, opportunities for teacher involvement and the effective dissemination of information are also essential. The dissemination of information is understood as the process of informing teachers, schools and society in general about new or revised curriculum ideas, documents or materials, so that they understand and accept innovation (McBeath, 1997). Figure 1.2 shows the dissemination of information about changes to the mathematics curriculum through the main types of steering documents, namely the curriculum, school plan, and guidelines for teachers.

Figure 1.2: Dissemination of main steering documents covering mathematics instruction, ISCED levels 1 and 2, 2010/11



Source: Eurydice. UK ([1]) = UK-ENG/WLS/NIR

Country specific notes

Czech Republic: The Manual for the development of school education programmes at basic schools and the System of building financial literacy in basic and secondary schools are considered to be guidelines for teachers.
Denmark: The national authorities develop and publish the *Fælles Mål* which provides central guidelines and objectives for mathematics education. This document is not defined as the curriculum in national regulations.
Cyprus: Figure 1.2 refers to ISCED 1. At ISCED 2, the curriculum, syllabus and school plans are officially published; special websites are developed; printed copies are distributed to schools; and central education authorities provide notes and implementation rules. In addition, textbooks are provided in accordance with the syllabus and school plans.
Luxembourg: At primary level, the curriculum is printed and distributed to schools. At secondary level, the curriculum is available on a specially developed website (www.myschool.lu).

Before starting the discussion of dissemination methods, it is important to consider the official status of steering documents in European countries. Documents that have an official status are usually published in the public or 'official journal' of the country. There is some form of official publication of the curriculum or other steering documents in all European education systems. Around half of all countries also have an official publication of teachers' guidelines, while school plans are officially published in approximately one third of European education systems. In Spain, both the national core curriculum and the curricula of the Autonomous Communities are officially published in the State Gazette and the Gazettes of Autonomous Communities.

([2]) More detailed information could be found at: http://visc.gov.lv/saturs/vispizgl/programmas/pamskolai/mat1_9.html

These days, the most common way of disseminating the curriculum and other steering documents covering mathematics teaching at primary and lower secondary level is by developing special websites. The curriculum is published on a designated Internet site in all European countries. Most European education systems also publish online guidelines for teachers. Subject syllabuses and school plans (or model examples) are also available on websites published by central authorities in about half of all European countries.

Websites usually belong to or are managed on behalf of the ministry of education or the country's main education research institute. Belgium (French Community), the Netherlands, the United Kingdom (Scotland) and Norway have a special website for the curriculum and other learning materials. In some countries, there are also regional websites, providing regional-level official documents (as, for example, in the case of the curricula of the Autonomous Communities in Spain).

Printed copies of the curriculum are distributed to each school in the majority of European education systems. In addition, printed guidelines for teachers are also sent to each school in almost half of the countries. Printed copies of syllabuses are distributed in Malta, the Netherlands, Liechtenstein and Turkey. Usually, printed copies are distributed to schools at the time of their publication. Some countries also distribute other types of materials to schools.

In approximately half of European education systems, central education authorities also provide notes and implementation regulations for the curriculum. About one third of countries issue guidance notes for teachers. Such additional information is less common for syllabuses and school plans.

1.2. Revising the mathematics curriculum and monitoring its effectiveness

The regular revision of the mathematics curriculum and the monitoring of teaching and learning are intended to help verify the relevance of learning objectives and ensure that the desired learning outcomes are being met. Subject content may also be adapted and improved. As the curriculum has compulsory status in almost all countries, any changes introduced are usually implemented gradually and, in some cases, more than two/three school years are needed before the implementation of new content or learning objectives is completed.

Main changes to the curriculum in the last decade

The raising of educational standards and, consequently, student achievement, is a constant objective of education reform. In all European countries, the mathematics curriculum has been revised during the last decade and, in the vast majority of countries, important updates have been introduced since 2007 (see Figure 1.3 for curriculum revision dates by level of education). One of the main reasons for the more recent updates was the inclusion of the learning outcomes approach, defined in a broad sense as the knowledge and skills that are needed to prepare a young person for personal well-being, social and working life (Psifidou, 2009). In the European qualifications framework (EQF), learning outcomes are defined as statements of what a learner knows, understands and is able to do on completion of a learning process; these are described in terms of knowledge, skills and competences ([3]). Curricula based on learning outcomes focus on the results of learning processes. Compared to traditional subject-based curricula, learning-outcome based curricula aim to be more comprehensive and flexible.

([3]) Recommendation of the European Parliament and of the Council of 23 April 2008 on the establishment of the European qualifications framework for lifelong learning. Official Journal of the European Union C 111, 6 May 2008, pp. 1-7.

Although the empirical evidence that the learning outcomes approach is a better curriculum planning mechanism than the aims/objectives approach is insufficient (Ellis and Fouts, 1993; Darling-Hammond, 1994), it is, however, possible to list potential advantages (March 2009, p. 50).

Learning outcomes:

- are more explicit statements about what students should be able to do;
- allow teachers more flexibility in planning their teaching;
- place less emphasis on the content to be covered and more emphasis on the skills/competences to be achieved;
- provide more concrete details about student performance for parents;
- enable teachers and school heads to be more accountable for student standards (maybe paraphrase to 'achievement'?);
- can address higher-order thinking skills;
- acknowledge different learning styles and forms of thinking.

The use of learning outcomes in curricula may also be related to new concepts of governance and quality management. It is argued that the formulation of learning outcome-based standards is a way of ensuring quality in education provision while at the same time granting more autonomy to providers to define learning programmes which meet the needs of their students (Cedefop, 2010).

A particular group of countries was driven to update the curriculum in order to meet the need for more individualised learning approaches; addressing the personal development needs of students and ensuring that specific assessment practices are aligned with the needed learning outcomes.

Several other reasons for introducing revisions to the mathematics curriculum were also given by European countries, including changes in learning content to incorporate cross-curricular links between subjects; the introduction of specific assessment targets, creating more flexibility in the learning process and facilitating the effective transition from one level of education to the next.

Due to the recent revisions, the content of the mathematics curriculum has been reduced in many countries. Furthermore, the related syllabuses have also been transformed from a list of specific mathematical concepts into an integrated system that develops problem-solving skills using mathematical principles. In addition, in Estonia, Greece, France, Italy, Portugal and the United Kingdom, the new curricula became more focused on cross-curricular links and the interaction of mathematics with philosophy, science, and technology. The view that mathematics subject content and skills serve as the basis for learning other school subjects has also become more widespread.

In **Estonia**, for example, the curriculum adopted in 2010 includes computing, numbers and algebra, measurement, and geometric shapes. Logic and probability topics are no longer specially highlighted in stage two (grades four to six) and are only introduced later on in grades seven to nine. Finally, some geometry theorems (for example Euclid's' theorem) are now excluded from the curriculum.

In **France**, the consecutive reforms during 2007-2008 modified the content of the mathematics curriculum reducing the content applicable to all students but reinforcing attention on both problem-solving and procedural skills. Nevertheless, in the upper secondary curriculum after the 2009 reforms, new content such as mathematical algorithms and probability were introduced and resource documents related to these new topics were developed by the education authorities.

In **Portugal,** after the reforms to the curriculum in 2008, the current programme became more explicit about what student performance should be in each of the mathematical topics and in the mathematics-related cross-curricular skills.

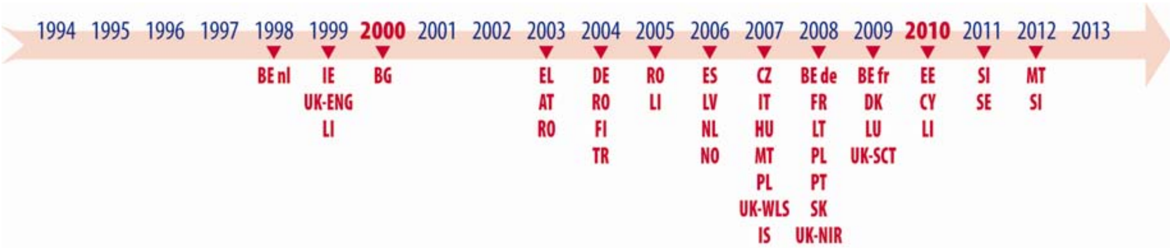
For instance, the present 'numbers' topic intends that students should develop a sense for numbers and understand numbers and operations; the 'algebra' section aims to develop their algebraic thinking; 'geometry' intends to develop geometric reasoning and visualisation; and finally the 'statistics' topic aims to develop students' statistical literacy.

In the **United Kingdom**, curriculum revisions in mathematics focus on skills and integrated learning. Specifically, in **England**, the new secondary mathematics programmes of study emphasise problem-solving, functionality and mathematical thinking, whereas the previous curriculum tended to be viewed as more content-focused. In **Wales**, the revised curriculum has reduced subject content and put increased focus on skills. In **Northern Ireland**, the structure of the curriculum has been revised with the aim of retaining the best of current practice while giving greater emphasis to elements such as 'Personal development and mutual understanding' and 'Thinking skills and personal capabilities'. Mathematics is one of six areas of learning, which are designed to be integrated, where appropriate, in order to make relevant connections for students across the different areas.

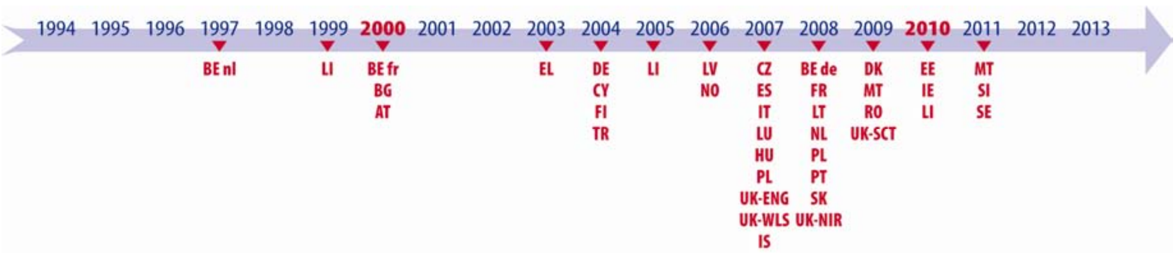
Finally, after the recent curriculum updates, in the majority of countries there is an improved connection between the knowledge acquired at school and students' personal experiences and problems in everyday life.

Figure 1.3: Latest revision and update of the mathematics curriculum, ISCED levels 1, 2 and 3

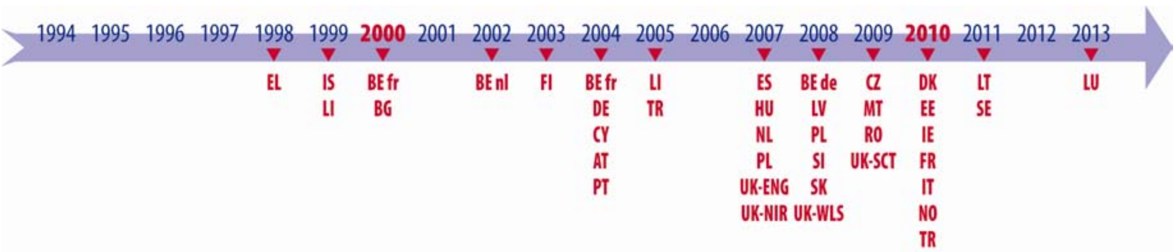
- **Primary education**



- **Lower secondary education**



- **Upper secondary education**



Source: Eurydice.

Country specific notes

Belgium (BE fr): The data show the education reforms in the French Community. The education programmes of the Free Religious Education Network were updated in 2005 for primary education, in 2000 for lower secondary education, and in 2008 for upper secondary education.
Slovenia: The updated curriculum for basic school (ISCED 1 and 2) will be implemented from the 2011/12 school year.

More specific assessment targets were introduced into the mathematics curricula of a number of countries, largely as a result of the impact of external examinations (Moreno, 2007) and the adoption of the learning outcomes approach. In addition, in the Netherlands and the United Kingdom (Scotland), for example, where a greater degree of autonomy is given to schools to determine teaching content and methods, specific assessment targets are the main instrument by which public authorities harmonise the evaluation of student performance. In Spain, another important change related to the definition of attainment targets which was introduced after the latest reforms in 2006 was the integration of the nationwide general evaluation of the education system with the regional diagnostic evaluation system (carried out in the different Autonomous Communities). The first is the responsibility of the Ministry of Education in collaboration with the Autonomous Communities and its main objective is to collect representative data (through national standardised tests) on the achievement of the targets (as defined in the curriculum) for the acquisition of basic competences.

Other factors that influenced not only mathematics teaching but also the general reform programme were the introduction of more flexibility in the implementation of study programmes and consistency between education levels.

For example, in secondary education in **Spain**, the Act on Education *(Ley 2/2006 Orgánica de Educación, 2006)* emphasises the importance of diversity, and ensures that different options and opportunities are available to meet the diverse needs of students. Any choices made should not prove irreversible or lead to an unavoidable exclusion and provision should reflect the competences and knowledge demanded by the 21st century society.

In **Estonia,** students may choose between two specially developed courses in mathematics. The new upper secondary school curriculum adopted in 2010 includes a narrow mathematics course with 8 modules (the course consists of 35 lessons of 45 minutes), and a wider mathematics course with 14 modules, which is considerably more flexible than the previous 2002 curriculum.

The new **Polish** Core Curriculum (with phased implementation) envisages direct continuity between the different levels of education, especially between lower and upper secondary levels. The general requirements are formulated in the curriculum in such a way that they refer to the same skill groups at every stage of education and specific requirements are not repeated. However, the study of some selected topics is extended during subsequent stages.

To assure the smooth transition between the different levels of education in mathematics education many programmes follow the 'spiral arrangement' model where all aspects of mathematical content and concepts build on each other and allow students to develop deeper understanding from one level to another.

In the **Czech Republic**, for example, the Framework Education Programme for Basic Education (FEP BE) is linked conceptually to the Framework Education Programme for Pre-primary Education (FEP PE) and is the basis for designing educational programmes for upper secondary education. It defines everything that is common and necessary for pupils in basic compulsory education, including education in the appropriate grades of multi-year upper secondary schools. It specifies the level of key competencies which pupils should achieve at the end of basic education, defines the content of education, the expected outcomes and curriculum, and includes compulsory cross-curricular subjects.

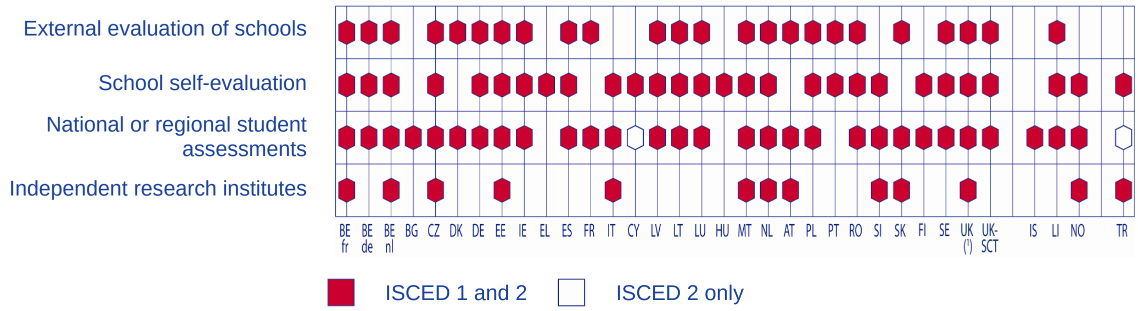
In the **United Kingdom** (**Wales**), the new learner-focused, flexible curriculum was introduced in 2008. In addition to the reduction in subject content, an increased emphasis was placed on skills, and a special focus put on continuity and progression by building on the foundation phase and linking effectively with the 14-19 years olds' learning pathways.

Evaluating the effectiveness of the curriculum implementation

Most countries attempt to assess the effectiveness of the implementation of the curriculum. This evaluation is carried out in diverse ways in the different European countries (see Figure 1.4). However, in the majority of countries, curriculum effectiveness is mainly evaluated through the **national student assessment** process. Standardised tests and centrally set examinations, where one of its objectives is to evaluate the implementation of the curriculum, are carried out in almost all analysed educational systems.

Specific surveys of how the curriculum is taught in schools are rare but this type of information is usually gathered as part of the general framework for the **external evaluation of schools**. However, the outcomes of **school self-evaluation** are the second most common source of data used by countries to evaluate the effectiveness of their curricula.

Figure 1.4: Sources of evidence for evaluating the curriculum, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

UK (¹) = UK-ENG/WLS/NIR

Country specific notes

Belgium (BE nl): These tests do not concentrate on the achievement of individual students but are used only for system-monitoring.
Iceland: School self-evaluation is obligatory but schools do not have to focus on the curriculum.

In evaluating curriculum effectiveness through the results of national or regional student assessment, countries consider not only the main trends in student results, but also the effects of, for example, the social background of students. They look for differences between regions and schools.

External evaluation of schools is carried out in almost two thirds of education systems and is typically performed by school inspectors but also, in some cases, by other national education organisations. In a few countries, this external evaluation includes the specific monitoring of the way the curriculum is being implemented in schools.

> In the **Czech Republic**, the implementation of the Framework Educational Programme and School Educational Programmes is monitored and evaluated by the Czech School Inspectorate. The monitoring of the curriculum is part of its regular inspections but, in addition, in 2010, it carried out one of its occasional thematic inspections which focused on mathematics literacy.

> The **Finnish** National Board of Education examines student performance in mathematics using a sample-based evaluation system. Approximately every ten years it also takes a sample of local curricula for analysis.

The external evaluation of schools in Lithuania and the United Kingdom assess teaching approaches and proposed means of improvement.

The National Agency for the Evaluation of Schools in **Lithuania** has concluded that most teachers still prefer teacher-centred instruction instead of student-centred learning. Very often, teachers cannot set clear and measurable goals and follow them during the teaching and learning process (NMVA, 2010).

Based on an evaluation carried out by Her Majesty's Inspectorate of Education (HMIE) in the **United Kingdom** (**Scotland**), it has been found that some primary school teachers lack confidence in certain aspects of mathematics and this 'inhibits an expansive approach to developing concepts and skills'. In secondary school, mathematics specialists have more exposure to the subject but use teaching approaches based on memorising basic algorithms without discussing different approaches or highlighting the relevance of learning for everyday life. In both cases, there is a strong emphasis on learning and teaching driven by assessment as opposed to a deeper understanding of concepts alongside appropriate assessment methods that confirm effective learning.

In a great number of countries, general school self-evaluation, including the evaluation of mathematics programmes is established by law and performed regularly with a given time frame. For instance, schools in Belgium (Flemish Community), the Czech Republic and Finland are obliged to have their own system of self-evaluation in place. In Estonia, all teachers and schools must prepare an annual self-evaluation report.

In **Portugal**, every school carries out self-evaluation within the scope of the Mathematics Plan II at the end of the year. It includes an evaluation of the strategies implemented, student performance in mathematics, and the development and implementation of the mathematics programme.

Finally, one third of countries use **independent research institutes** to evaluate various aspects relating to the teaching of curriculum programmes and student assessment.

In **Belgium** (**French Community**), within the framework of a pilot project, the University of Liège will compare two external evaluations of the award of the certificate for secondary education and validate the threshold of success in four domains, including, mathematics.

In **Estonia**, the Curriculum Development Centre at the University of Tallinn published a study entitled '*Systemic person-oriented study of child development in early primary school*' (Toomela, 2010). The study covers a range of topics including the further development of the mathematics curriculum and its teaching.

The **Austrian** Parliament has established the *Bundesinstitut für Bildungsforschung, Innovation und Entwicklung des österreischischen Schulwesens*-BIFIE (Federal Institute for Education Research, Innovation and Development of the Austrian School System). It provides advice during the implementation of crucial reforms in education policy as well as summarising the results of national education research at regular intervals and publishing the information in a national education report (4).

In **Slovenia**, the Council for Evaluation in Education coordinates the evaluation of education programmes from pre-school to compulsory and upper secondary education. It specifies the strategy and methods of evaluation and also identifies fundamental evaluation issues. The Council also monitors the progress of evaluation studies and reports to the National Council of Experts and to the Minister. Evaluations are mainly carried out by the national Educational Research Institute (5) but some other research institutions are also involved.

(4) More information on the BIFIE can be found at: http://www.bifie.at/die-kernaufgaben

(5) More information on the Slovenian Education Research Institute can be found at: http://www.pei.si/pei_english.aspx

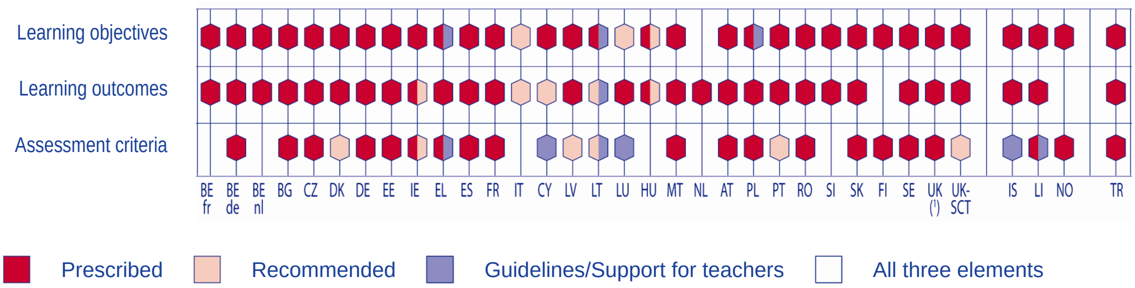
1.3. Learning objectives and mathematics content and competences in the curriculum

Learning objectives

Learning objectives and outcomes are important parts of the learning process. Learning objectives are general statements concerning the overall goals, ends or objectives of teaching, while learning outcomes are defined in more concrete terms. Learning outcomes are concerned with the achievements of the learner rather than the objectives of the teacher. Learning objectives are usually expressed in the aims of a module or course, while learning outcomes are usually expressed in terms of what the learner is expected to know, understand and be able to do on completion of a level or module (Harey, 2004). On the relationship with learning objectives, Adam (2004, p. 5) further explains that learning outcomes can take many forms and can be broad or narrow in nature. There is often some confusion between learning outcomes and learning objectives or aims, and certainly many regard learning outcomes and objectives as the same thing and use the terms synonymously. The important distinction is that learning objectives are associated with teaching and teachers' objectives whilst learning outcomes are concerned with learners' achievements.

As previously discussed, integrating learning outcomes in the curriculum planning process was one of the goals driving recent reforms in mathematics education. Currently, both learning objectives and learning outcomes are usually prescribed in European countries.

Figure 1.5: Objectives, outcomes and assessment criteria in the mathematics curriculum and/or other mathematics steering documents, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

UK (¹) = UK-ENG/WLS/NIR

Explanatory note

Assessment criteria apply only to in-class assessment by teachers and not to general national tests or assessments.

Country specific notes

France: Learning materials are recommended only for ISCED 2.
Hungary: Learning objectives and outcomes are recommended in the national framework curricula and prescribed in the National Core Curriculum and local curricula.

Learning objectives are both prescribed and recommended in Hungary where the National Core Curriculum (NCC) and local curricula prescribe learning objectives. The learning objectives in the NCC are expressed in terms of competences and attitudes, while local curricula express learning objectives in terms of knowledge and skills. In addition, learning objectives are recommended in accredited framework curricula.

Learning objectives and outcomes are also expressed as broad guidelines in the support materials for teachers in Greece, Lithuania, Poland and Turkey.

> **Lithuania** provides methodological recommendations regarding both learning objectives and outcomes in mathematics.
> **Poland** publishes official commentaries to the mathematics curriculum which include learning objectives.

Learning objectives and outcomes are only recommended in Italy in the official documents entitled *'National indications for learning objectives'* (for upper secondary school) and *'Indications for the curriculum'* (primary and lower secondary). These contain general descriptions of the main learning aims and expected outcomes at the various stages of education. On that common base, schools are supposed to define actual curricula for their own students in the various subjects. Luxembourg recommends learning objectives, but prescribes the learning outcomes for study programmes. In contrast, in Hungary learning objectives are prescribed, while learning outcomes are only recommended.

In order to ensure effective schooling, learning objectives and outcomes, as defined in the curriculum, need to be aligned with the approaches to teaching and assessment used in the classroom (Elliott, Braden, & White, 2001; Webb, 1997, 2002; Roach et al., 2009).

Assessment (which is covered in more detail in Chapter 3) is a prominent component of the entire teaching and learning process (McInnis and Devlin, 2002). For many students, and even teachers, assessment requirements tend to define what is learnt. Moreover, changing assessment systems and testing can be a powerful tool when implementing education reforms (Black, 2001). Therefore, when introducing expected learning outcomes in the curriculum, clear alignment with the assessment of knowledge and skills should be considered (Marsh, 2009).

Mathematics assessment criteria are prescribed in two thirds of European countries. However, they are only recommendations in Denmark, Portugal and the United Kingdom (Scotland). Luxembourg provides broad guidelines and support for teachers in this area.

In **Greece**, assessment criteria are published in the Official Gazette (303/13-3-2003), while additional guidelines and support for teachers are explained in the circular letters of the Ministry of Education, Lifelong Learning and Religious Affairs.

Lithuania issues recommendations on assessment methods for mathematics, while broad guidelines are described in the curriculum.

In the **United Kingdom** (**England**, **Wales** and **Northern Ireland**), in addition to the assessment criteria (levels of attainment, etc.), statutory assessment and reporting arrangements are also prescribed.

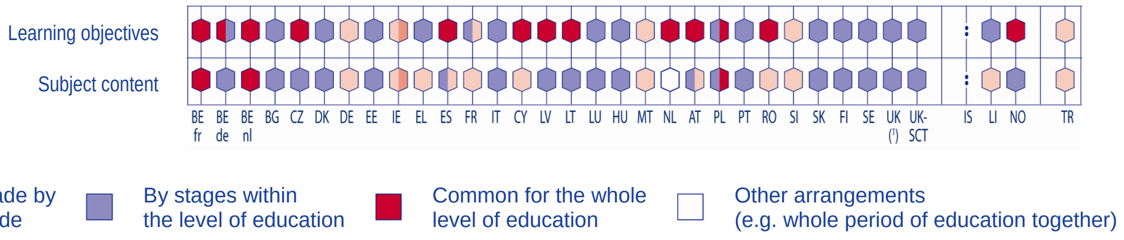
Belgium (French and Flemish Communities), Italy, Hungary and the Netherlands do not specify criteria related to in-class mathematics assessment.

Structure and progression in learning objectives and subject content

In the majority of countries, the learning objectives and content of mathematics programmes are set down either for each stage or cycle within the same level or for the entire level of education. Only in Germany, France, Malta, Slovenia and Turkey are both the objectives and content defined for each grade. In Belgium (German-speaking Community), the Czech Republic, Spain, Cyprus, Latvia, Lithuania, Austria and Romania are the learning objectives defined in the curriculum for the entire educational level while the content of the educational programmes are established either for each grade or for each stage within each education level.

In **Cyprus**, the curriculum objectives are developed in a continuum of eight scales from pre-school to upper secondary school. Each scale is broken down into attainment targets and some targets appear in consecutive scales to ensure the consistency of the syllabus through each grade.

Figure 1.6: Structure and progression in learning objectives and subject content, as prescribed in mathematics steering documents, ISCED levels 1 and 2, 2010/11



Source: Eurydice. UK (¹) = UK-ENG/WLS/NIR

Country specific note

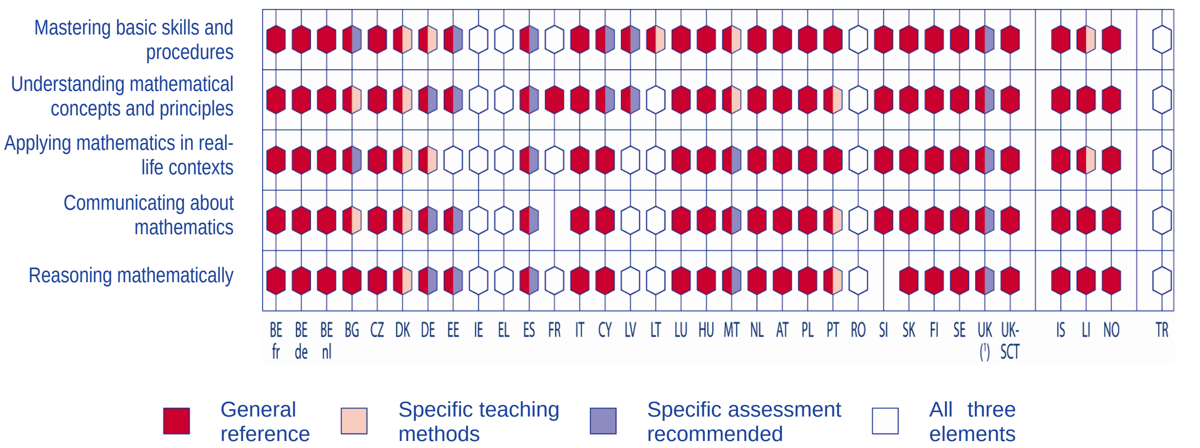
Hungary and **Finland**: The central framework curriculum normally defines the common targets and content for stages or levels of education while local curricula specify aims and content for every grade.

Learning content is distributed between different stages of varying duration. In Estonia, basic school (grades one to nine) is treated as a single structure but, for the purposes of the curriculum, it is divided into three stages of three years. Similarly, in Poland, mathematics education at primary level is divided into three stages: the first stage covers only the first grade, the second stage covers grades two and three, and the third stage from grades four to six. Some other countries define curriculum content for two-year stages throughout the entire period of schooling (e.g. Lithuania); Norway specifies mathematics competence targets for grades 2, 4, 7, 10, 11, 12 and 13.

Skills and competences in the mathematics curriculum

To ensure that students acquire the essential mathematical skills and competences, countries incorporate these requirements into their curriculum or into other mathematics steering documents. Figure 1.7 examines five key areas of mathematical skills, making a distinction between general references to particular skills in steering documents and where there are more specific references to skills in relation to teaching methods and/or assessment procedures.

Figure 1.7: Skills and competences in the mathematics curriculum and/or other mathematics steering documents , ISCED levels 1 and 2, 2010/11



Source: Eurydice. UK (¹) = UK-ENG/WLS/NIR

Country specific notes

Ireland: For secondary education, schools are required to regularly evaluate students' progress using a range of assessment approaches. No specific assessment methods are recommended.
Spain: The curriculum includes assessment criteria for each subject and teachers must assess students' acquisition of competences according to these criteria.
Cyprus: Refers to ISCED 1. At ISCED 2, there are only general references to all elements, except applying mathematics in real-life contexts, which is not mentioned.
Malta: Specific assessment using checklists in core competences is recommended at Primary level years 1 to 3.
United Kingdom (ENG/WLS/NIR): Guidance on assessment is non-statutory (i.e. not a legal duty) rather than recommended.

The key mathematical competences are mentioned at least in general terms in the curriculum or other steering documents in almost all European countries. In nearly half of the education systems, only general references are made. However, in some countries (Denmark, Portugal and Liechtenstein) suggestions are also given for specific teaching methods to be used when teaching these skills. Moreover, in Greece, Romania and Turkey, both specific teaching methods and recommendations for student assessment are mentioned with respect to all five areas of skills.

Overall, the investigation did not reveal very many differences between the areas of mathematical skills in that each of them was specifically mentioned in about the same number of European countries. However, both specific teaching methods and assessment were suggested more often in relation to the area 'applying mathematics in real life contexts'.

Mathematics subject content

According to research, the curriculum and other steering documents have a powerful influence on what students learn (Valverde et al., 2002; Thompson and Senk, 2008). Furthermore, research based on international student achievement data shows that countries with a similar mathematics curriculum tend to have similar responses to questions on students' competences in mathematics (Wu, 2006). The difference in the emphasis on certain subtopics, or whether the topic was included in the mathematics curriculum, could be associated with different performance patterns (Routitsky and Zammit, 2002; Zabulionis, 2001). Therefore, it is important to explore how the curriculum is organised and what topics are covered.

National regulations on the subject content of mathematics programmes as presented in the Annex 1 confirm that almost all topics covered by the area **numbers** are present in all European countries in both primary as well as secondary level. Bulgaria, Germany, Lithuania, Slovenia, Slovakia, Finland and Norway split the topics between the two levels, generally including issues like 'represent whole numbers' or 'basic mathematics operations' during the first years of studying, and leaving other topics (see Annex 1) to the later years of primary or even secondary education. In France and Italy, all the topics analysed in the 'numbers' section are in the study programme but some, such as, 'estimation of computations by approximating the numbers involved' or 'operations with fractions and decimals' are only studied at a basic level during the first years and are covered in more depth during secondary level.

The area **geometry** is covered by all educational programmes but the depth of study of topics in this area varies around Europe. Learning the basic geometrical concepts (e.g. 'point', 'line segment', 'line' or 'angle') is mentioned in all national study programmes. Measuring or estimating the size of given angles, the length of lines, perimeters, areas and volumes of geometric shapes are processes mentioned in the majority of programmes. However, countries such as Bulgaria, Germany, Lithuania, Hungary, Austria, Slovakia, Finland Sweden and Liechtenstein devote curriculum time to these processes mainly in secondary education.

More advanced geometry topics such as, 'ordered pairs', 'equations', 'intercepts', 'intersections', and 'gradient to locate points and lines in the Cartesian plane' are only included at secondary level with the exception of the United Kingdom (England, Wales and Northern Ireland), Iceland, Turkey and partially studied in Italy.

In general, the three **algebra** topics are found almost exclusively in secondary education. 'Find sums, products, and powers of expressions containing variables' and 'evaluate equations/formulas given values of the variables and solve problems by using them' are present in all the education programmes in secondary education. Few countries include any of these topics at primary level – only Estonia, Greece, United Kingdom (England, Wales and Northern Ireland) and Iceland. 'Extending numeric, algebraic and geometric patterns' and 'sequences using numbers, words, symbols, or diagrams' are more equally represented in both levels of education but problems related to the 'finding of missing terms' and 'generalising the pattern relationships between terms' occur more frequently at secondary level.

Finally, the fourth main area of mathematics, **data and chance**, is also widely represented in European programmes of study. Some of the basic competences like 'read data from tables, pictographs, bar graphs, pie charts, and line graphs' are included from primary education with the exception of Belgium (Flemish Community), Bulgaria, Luxembourg, Romania and Sweden. Twelve countries include the study topics related to knowledge of the 'organisation and display of data using tables, pictographs, bar graphs, pie charts, and line graphs' only at secondary level.

Judging probability and predicting future outcomes using data from experiments are the topics that are least frequently included, but when they are, it is usually at secondary level. Only a few countries (Ireland, Spain, Slovenia, the United Kingdom, Iceland and Turkey) include probability both at primary and secondary level. On the other hand, Belgium (Flemish Community), Bulgaria, the Czech Republic, Germany, Cyprus and Finland do not include probability or prediction in the compulsory content of their mathematics curriculum, although these questions are dealt with in conjunction with other topics.

1.4. Taught time devoted to mathematics teaching

Recommended minimum taught time for mathematics

The recommended taught time for mathematics (curriculum time during which students are taught mathematics) in primary and secondary schools is an important proxy that helps to explain the relative importance of the subject compared to others in the curriculum.

European countries organise their annual taught time differently during primary and lower secondary education. The official amount of taught time allocated to particular subjects does not always provide an accurate reflection of the time students spend on a subject as, in many cases, schools have the right to allocate additional time to specific subjects or they may have complete autonomy in the overall distribution of taught time (Eurydice, 2011). However, the overall timetable is less intensive at the beginning of primary education (generally for the first two years), then steadily increases through compulsory education, with a significant increase in hours at lower secondary level.

Where recommendations for taught time are given for each subject, mathematics instruction accounts for between 15 % and 20 % of total taught time in primary education, coming second only to the language of instruction. Portugal is the only country where the taught time allocated to mathematics is more than 20 % of the total learning time in primary education.

Figure 1.8: Percentage of the recommended minimum taught time dedicated to mathematics compared to the total taught time during full-time compulsory education, 2009/10

	Level of education		Total for compulsory education
	Primary education	Compulsory secondary education	
BE fr	HF	9.5	HF
BE de	HF	9.4	HF
BE nl	HF	HF	HF
BG	15.9	11.8	13.1
CZ	16.9	12.3	14.6
DK	15.3	12.9	14.5
DE *(Grundschule + Gymnasium)*	10.9	11.4	11.2
DE *(Grundschule + Hauptschule)*	10.9	20.7	16.8
DE *(Grundschule + Realschule)*	10.9	14.1	13.1
EE	15.2	13.5	14.6
IE	16.1	7.0	10.6
EL	15.2	11.4	13.8
ES	10.7	9.1	10.0
FR	17.2	17.4	17.3
IT	HF	19.0	HF
CY	18.9	11.6	15.6
LV	17.0	15.5	16.4
LT	16.4	12.0	13.4
LU	19.0	10.0	15.4

	Level of education		Total for compulsory education
	Primary education	Compulsory secondary education	
HU	17.6	12.3	13.8
MT *(Primary+Lyceum)*	19.2	13.5	16.7
MT *(Primary+Secondary)*	19.2	14.3	17.1
NL	HF	HF	HF
AT *(Volksschule + Allgemeinbildende Höhere Schule)*	17.8	13.9	15.4
AT (*Volksschule + Hauptschule + Polytechnische Schule)*	17.8	13.8	15.3
PL	HF	10.6	HF
PT	21.8	9.2	16.9
RO	14.0	14.0	14.0
SI	17.2	12.6	15.5
SK	17.5	14.3	15.7
FI	17.5	11.8	14.4
SE	13.5	13.5	13.5
UK-	HF	HF	HF
IS	15.1	13.5	14.6
LI *(Primary+Gymnasium)*	18.2	13.8	16
LI *(Primary + Oberschule/ Realschule)*	18.2	14.8	16.5
NO	17.2	11.0	15.0
TR	13.3	20.0	15.7

Source: Eurydice.

Explanatory note

HF: Horizontal flexibility. Curricula only specify the subjects and the total taught time per year, without specifying the time to be allocated to each one of them. Schools/local authorities are free to decide how much time should be assigned for compulsory subjects.

Country specific note

Spain: The indicated taught time for mathematics corresponds only to the minimum taught time prescribed in the national core curriculum. The Autonomous Communities are accountable for between 35-45 % of the total taught time and allocate additional time to mathematics.

In Spain, mathematics in primary education covers around 16 % of the national core curriculum and 10 % of the total recommended timetable for this level. However, the compulsory curriculum adopted by the central level in Spain represents between 55 % and 65 % of the total taught time; the Autonomous Communities are accountable for the remainder of the timetable and can allocate additional time to mathematics, although they cannot devote all the remaining time to only one subject. In Luxembourg and Malta, mathematics is the subject with the highest number of dedicated hours during primary education. This is due to the fact that the taught time recommended for the learning of official languages is split in two categories, defined as language of instruction for the first language and foreign languages for the others.

The official breakdown of taught time for compulsory subjects is very different in primary education to compulsory general secondary education. At secondary level, the proportion of time allocated to the language of instruction and mathematics decreases, while the time given over to natural and social sciences and foreign languages increases in nearly all countries. However, in some countries, the absolute number of hours dedicated to mathematics is stable. In compulsory secondary education, mathematics accounts for between 10 % and 15 % of the overall timetable. In Germany *(Hauptschule)*, France, Italy and Turkey, however, mathematics takes up a higher percentage of the total taught time, reaching the 20 % mark.

In primary education mathematics is taught for 110-120 hours per year, on average, but important differences exist between countries. In Germany, Greece, France, Austria, Liechtenstein and Turkey, there is a tendency to have the same number hours per year during the entire primary level. The highest average number of hours per year for mathematics (137 hours) can be found in these education systems (excluding Turkey). In a second larger group ([6]) of countries, the annual taught time increases with the age of students, starting from 72 or 75 hours in Bulgaria and Lithuania for the first grade and growing till the last grade of primary level. A third approach used in some countries is to have a decreasing number of recommended hours for mathematics during primary education. In these cases, generally during the first two years of primary education, pupils have between 150-160 hours per year (up to 216 in Luxembourg and 252 in Portugal) but the figure decreases in the later years of primary education.

During compulsory secondary education, within their recommended timetables most countries allow some hours to be allocated flexibly between subjects. In general, schools can distribute these hours between the core subjects or provide special cross-curricular activities or reinforcement lessons. Moreover, in Belgium (Flemish Community), the Netherlands, Sweden (within each subject) and the United Kingdom, schools have complete freedom to determine the time allocation for all subjects throughout the entire period of compulsory education.

([6]) Bulgaria, Estonia, Ireland, Latvia, Lithuania, Romania, Slovenia, Slovakia, Finland.

Figure 1.9: Recommended minimum taught time for mathematics during full-time compulsory education, 2009/10

● Flexible time

| x Number of hours distributed between various school years

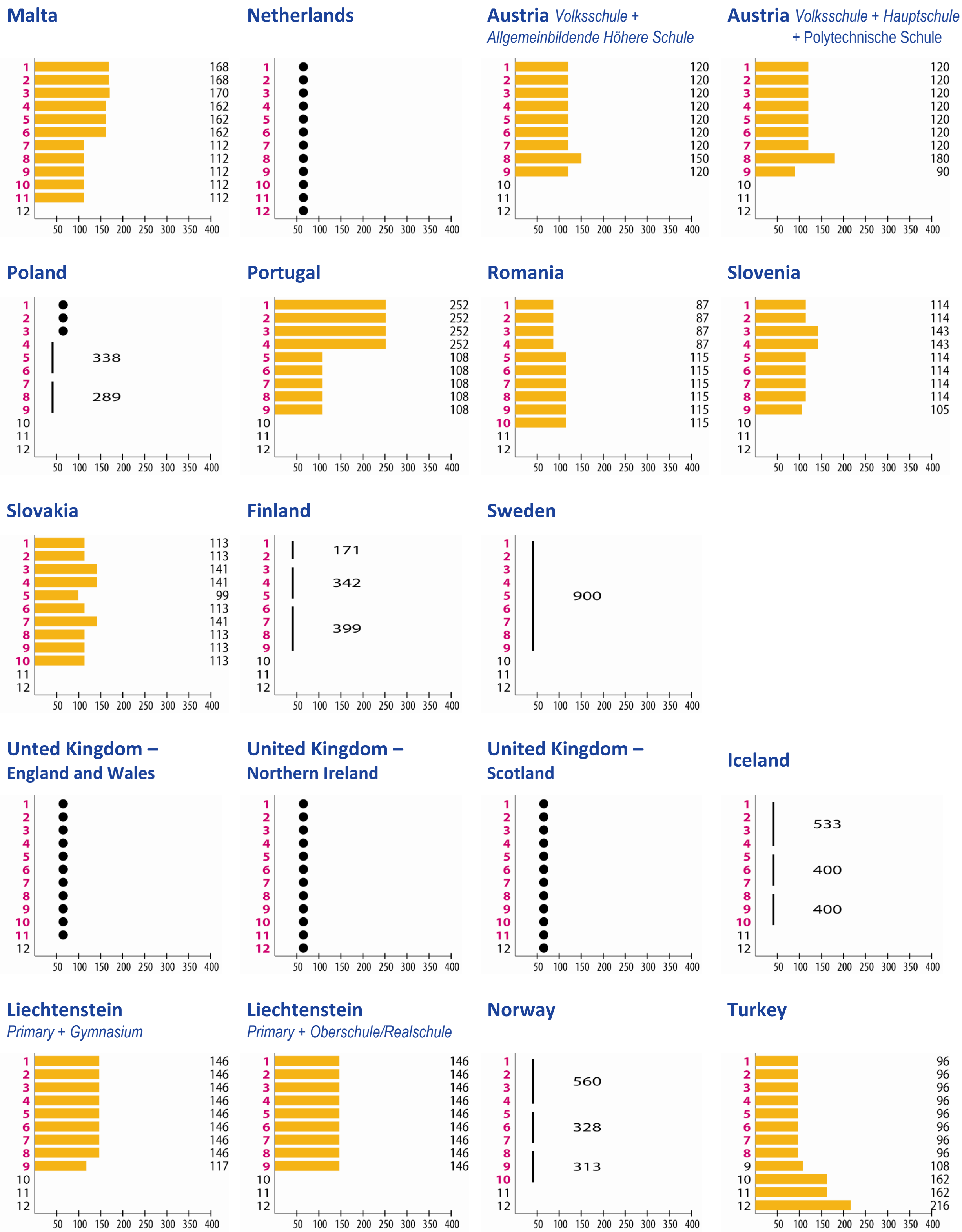
Horizontal axis: Number of hours per school year

Vertical axis: Grades

n: Compulsory education

Source: Eurydice.

Figure 1.9: (continued): Recommended minimum taught time for mathematics during full-time compulsory education, 2009/10



● Flexible time

❙ x Number of hours distributed between various school years

Horizontal axis: Number of hours per school year

Vertical axis: Grades

n: Compulsory education

Source: Eurydice.

Country specific notes

Spain: The indicated taught time for mathematics corresponds only to the minimum taught time prescribed in the national core curriculum. The Autonomous Communities are accountable for between 35-45 % of the total taught time and allocate additional time to mathematics.
Italy: In grades 6-7-8, the total amount of 198 hours refers to the whole subject area of 'Mathematics, physical and natural sciences'. The time devoted to mathematics can be estimated at about 132 hours for each year but some flexibility exists. In grades 9-10, the taught time devoted to mathematics depends upon the chosen branch. It can be estimated at between 99 and 132 hours for both years.
Poland: Data for grades 7-9 relate to the new timetables introduced gradually since 2008. Data for grades 4-6 relate to the old timetable. However, it is already decided that taught time dedicated to mathematics instruction in grades 4-6 will be the same as in grades 7-9 and will equal 289 hours.

Actual distribution of taught time between mathematics topics

International surveys provide some additional information on the actual time dedicated to various mathematics topics in the classroom. This section briefly presents TIMSS 2007 data on how the taught time for mathematics is distributed across several different content areas, according to teachers' reports. In addition, students' most frequent activities in mathematics lessons, as reported by teachers, are discussed. The numerical data is from Mullis et al. (2008, p. 196).

At the fourth grade, the mathematics content areas analysed by TIMSS were 'number', 'geometric shapes and measures', and 'data display'. At the fourth grade, on average in the participating countries from the European Union ([7]), teachers reported devoting over half (54 %) of mathematics taught time to the 'number' content area (e.g. computation with whole numbers, fractions, decimals and number patterns), about a quarter (23 %) to 'geometric shapes and measures' (e.g. two- and three-dimensional shapes, length, area and volume), 15 % to 'data display' (e.g. reading, making, and interpreting tables and graphs), and 9 % to other areas. Sixty per cent or more instruction time was devoted to the 'number' content area in Hungary, the Netherlands, Slovakia and Norway. On the other hand, in the Netherlands geometry topics were taught for the least amount of time (only 15 %). All these countries show an approach consistent with the recommendations in the curriculum for each stage as reported in Section 1.3 where the 'numbers' area features strongly in primary education and the 'geometry' area is largely found at secondary level.

At the eighth grade, the content areas analysed in TIMSS were 'number', 'algebra', 'geometry', 'data and chance'. On average in the participating EU countries, teachers reported devoting 23 % of mathematics instructional time to 'number' (e.g. whole numbers, fractions, decimals, ratio, proportion and percent), 31 % to 'algebra' (e.g. patterns, equations, formulas and relationships), 28 % to 'geometry' (e.g. lines and angles, shapes, congruence and similarity, spatial relationships, symmetry and transformations), 14 % to 'data and chance' (e.g. reading, organising and representing data, data interpretation and chance), and 5 % to other areas. The 'number' topics took 35 % or more instructional time in Slovenia, Sweden and the United Kingdom (Scotland). In contrast, the 'number' content area was taught for less than 20 % of the time in Bulgaria, Italy and Romania. Instead, teachers in these countries reported stronger emphasis on 'geometry' (more than 30 % of instructional time). Less than 20 % of instructional time was devoted to 'algebra' in Norway, while in Bulgaria, the Czech Republic, Italy, Cyprus and Lithuania the figure was over 30 %. The 'data and chance' content areas had less emphasis in Bulgaria, the Czech Republic and Cyprus (less than 10 % of instructional time) (see Mullis et al. 2008, p. 197).

TIMSS 2007 also gathered data on how frequent some mathematics activities were carried out in the classroom. The activities analysed for both fourth and eighth grade students were 'practice adding,

([7]) Here and further the Eurydice calculated EU average refers only to the EU-27 countries which participated in the survey. It is a weighted average where the contribution of a country is proportional to its size.

subtracting, multiplying, and dividing without using calculator' and 'work on fractions and decimals'. Other categories differed between grades. At the fourth grade, 'write equations for word problems', 'learn about shapes such as circles, triangles, rectangles, and cubes', 'measure things in the classroom and around the school' and 'make tables, charts, or graphs' were considered. At the eighth grade, the activities were more complex, namely 'write equations and functions to represent relationships', 'use knowledge of the properties of shapes, lines, and angles to solve problems', and 'interpret data in tables, charts, or graphs'.

According to teachers' answers, the most frequent activity for fourth grade students in mathematics was 'operations with whole numbers'. On average in participating EU countries, 87 % of fourth grade students had teachers who reported that students frequently 'practiced adding, subtracting, multiplying and dividing without a calculator'. About 30 % of fourth grade students had teachers who reported asking them to 'write equations for word problems' and 17 % reported that students worked on fractions and decimals in at least half of the lessons or more. Learning about shapes such as circles, triangles, rectangles, and cubes and making tables, charts or graphs were less common. However, measuring things in the classroom and around the school was the least common type of activity according to TIMSS. Teachers of only 3 % of fourth grade students reported this activity in around half of lessons.

At the eighth grade, teachers reported spending somewhat less time on operations with whole numbers, and more time working on fractions and decimals than at the fourth grade. According to teachers, on average in the EU, 61 % of eighth grade students frequently 'practiced adding, subtracting, multiplying and dividing without using a calculator'. About half of the eighth graders (48 %) often worked on fractions and decimals, according to their teachers' reports. With respect to 'operations with whole numbers', Norway reported only 9 % of students doing this activity frequently and therefore stood out as the exception. At the other end of the scale, Romanian teachers reported that 93 % of eighth grade students worked with whole numbers – this was more often than any other European country (for exact country numbers see Mullis et al. 2008, p. 283)

Using knowledge of the 'properties of shapes, lines and angles to solve problems' was a common activity for 40 % of European eighth grade students, according to their teachers. However, less than 15 % of students were often engaged in such activities in Sweden, the United Kingdom (England and Scotland) and Norway. In contrast, more than 70 % of eighth grade students 'used geometrical properties to solve problems' frequently in Bulgaria, Italy and Romania.

According to teachers, 'interpreting data in tables, charts or graphs', was a frequent activity for approximately 11 % of eighth grade students in EU countries.

1.5. Textbooks and learning materials in mathematics

This section reviews existing practices across Europe in the production, use and monitoring of textbooks and other learning materials for mathematics teaching. Textbooks and materials may influence teachers' beliefs about mathematics (Collopy, 2003) or knowledge of the subject (Van Zoest and Bohl, 2002) and, in this way, affect their interpretation of the written curriculum. It is therefore important to align teaching materials with the curriculum. Schools are often overwhelmed with information from textbook publishers claiming their materials comply with the benchmarks and standards laid down in steering documents. However, some deeper analysis shows that learning materials may suffer from a lack of consistency and focus (Kulm, Roseman and Treistman, 1999).

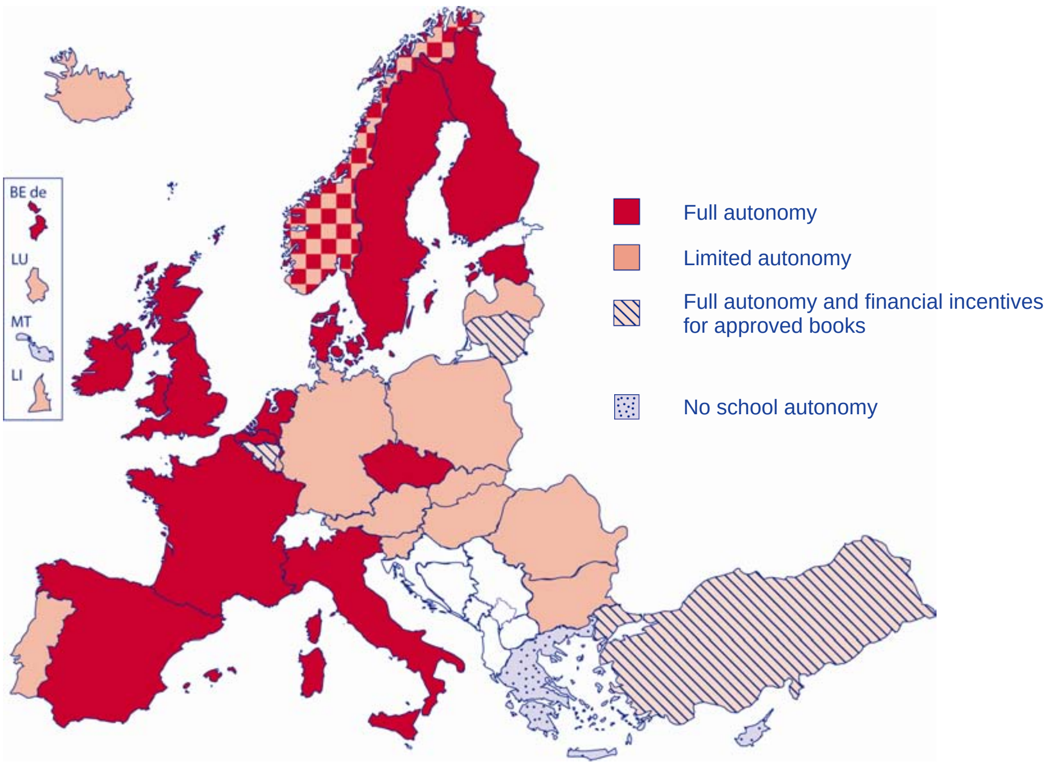
Degree of school autonomy in the choice of mathematics textbooks

In general, schools have some degree of autonomy in the choice of mathematics textbooks (see Figure 1.9). Most countries indicate full autonomy which means that schools are free to choose from all available textbooks. In Norway there is local variation due to local autonomy and school responsibility; hence it combines limited and full autonomy.

One third of countries have limited autonomy and schools have to choose either from a pre-determined list (for example Austria, Bulgaria, Liechtenstein, Latvia, Poland, Romania, Slovenia, and Slovakia) or are free to choose from all available textbooks previously approved by the Ministry of Education, as is the case in Portugal. Luxembourg has a mixture of these two approaches to limited autonomy. Schools are limited to one specific authorised mathematics textbook in only three countries, namely Cyprus, Greece and Malta and, in the latter, the prescribed textbooks are also distributed free of charge.

> In **Iceland**, schools have limited autonomy as they choose the textbooks from those offered by the National Centre for Educational Materials. The Centre is also responsible for providing learning materials free of charge to all students in compulsory schools. In addition, Icelandic schools receive funding to buy learning materials not provided by the Centre. The amount allocated depends on the number of students.

Figure 1.10: Levels of autonomy for choosing mathematics textbooks, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

Country specific note

Belgium (BE fr): Financial support to schools is granted only for the purchase of approved textbooks for primary and the first level of secondary education and educational software for pre-primary, primary and secondary education.

In United Kingdom (Scotland), the use of textbooks is at the complete discretion of individual schools and it is never stated that the use of textbooks is essential. Whilst most schools have a core textbook to support the learning of mathematics, a number of schools use a wide range of resources to provide the best support for learning.

In some countries, the choice of textbooks and learning materials can be influenced by financial mechanisms.

For instance, schools in **Lithuania** are free to choose from all available textbooks, however, if the textbook is not registered in the Ministry of Education and Science's Textbook Database, less funding is allocated to the school budget for other learning materials. A similar situation occurs in **Belgium** (**French Community**) where a subsidy is granted to schools which acquire textbooks approved by the French Community. In **Turkey**, schools are also free to choose textbooks, but those written and published by the Ministry of National Education are given to students free of charge. The Board of Discipline and Education certifies textbooks which are then usually chosen by schools as they are considered to be reliable.

Four countries provide specific financial incentives such as subsidies and loans for parents for the acquisition of textbooks.

In **Austria** and **Hungary**, only textbooks from a pre-determined or recommended list can be subsidised by the State.

Slovenia also offers book loans to reduce the financial burden on parents. Students may choose to borrow textbooks from textbook banks that operate in schools. The Ministry of Education and Sport earmarks funds for textbook loans for all students and discourages the use of other learning materials, thus reducing related costs.

In **Spain**, the Ministry of Education and the Autonomous Communities annually offer some grants to help families facing up the cost of textbooks. In some Autonomous Communities, apart from the aforementioned aids, there are also programmes to get free textbooks. In these programmes, the education authorities are the textbooks' owners and they lend them to pupils.

Production/development of textbooks

In the vast majority of countries, there is a free market for textbooks and many publishers design and produce them. In Cyprus, Iceland and Turkey national centres and institutes develop textbooks.

Some countries publish lists of textbooks approved by the relevant national authorities. While a few countries adopt regulations or guidelines which stipulate all the conditions that textbooks must meet (e.g. Bulgaria, Estonia and Latvia), several other countries specify only general criteria that textbooks must meet in order to be used by schools or be included on the approved list.

For instance, the Ministry of Education, Youth and Sport in the **Czech Republic** publishes an approved list of textbooks and teaching texts on its website. At the same time, schools may also use other textbooks if these comply with the educational objectives set down in the Education Act, other legislation, or educational programmes, and if their structure and content meet educational and didactical principles. The head of school is responsible for guaranteeing that these conditions are met when making the final decision on the choice of textbooks.

The basic general conditions for textbooks are also stated in **Lithuania**. The textbooks registered in the Textbook Database must meet minimum requirements – be democratic, cover a part of the curriculum and contain additional methodological tools.

Romania and Hungary specifically use tendering as a means of selecting textbooks. In Romania, the National Assessment and Evaluation Centre organises a nationwide tender process every five years. In addition, in 2008, the Ministry of Education, Research, Youth and Sport issued a specification for textbooks for pre-university education levels which sets down eight main quality criteria that must be met including complying with the curriculum and being non-discriminatory. The selected textbooks receive funding to cover printing costs. Schools must choose from a pre-determined list of titles. Whilst

Romanian schools have limited autonomy to choose textbooks, Hungary grants full autonomy. Nonetheless, Hungary also occasionally makes grants available through a tendering process for developing textbooks and learning materials.

National authorities in Greece, Latvia and Lithuania supervise the textbook development process and focus on particular stages. For instance, Greece supervises the design and production process while the State Education Centre in Latvia prepares a list of reviewers and publishers and chooses two reviewers for each book. The Education Development Centre of the Ministry of Education and Science in Lithuania is responsible for monitoring and evaluating the quality of textbooks as well as for encouraging innovation. The Centre also organises the evaluation of other learning materials on a regular basis in order to provide independent and professional information about the qualities of textbooks for consumers.

In some countries, the regulatory framework distinguishes between the provision of textbooks and the provision of other learning materials. This is mostly the case in countries where there are many publishers and full school autonomy to choose from all textbooks on the market, and where national institutes mainly concentrate on supporting the use of learning materials.

National institutes in Austria, Belgium (French Community), Denmark and Spain facilitate and support the use of learning materials. An educational portal ([8]) initiated by the national authorities in Denmark provides learning materials, services and resources online. The website currently encompasses information for teachers and pupils of primary schools, upper secondary schools, institutions for vocational training and teacher training colleges. Likewise, Spain has a website dedicated to curriculum and complementary resources for different subjects including mathematics and a separate website ([9]) for the dissemination of studies and reports on education and the publication of educational resources.

Monitoring and reviewing consistency between curricula and textbooks

Education authorities in the majority of countries report that they monitor and review the consistency between mathematics textbooks/learning materials with the mathematics curriculum or other regulatory documents (see Figure 1.10). It is worth mentioning that both groups of countries – those which monitor and review and those which don't – include countries that allow schools full, limited or no autonomy in the choice of textbooks and learning materials.

Professional reviews are a standard part of the textbook development process in countries such as the Czech Republic, Denmark, Estonia, Hungary and Latvia. The official approval of textbooks and teaching texts by the Czech Ministry is based on the expert opinion of at least two independent professional reviewers. Publishers in Estonia must find at least two reviewers; one of them must be an educator and the other a mathematics teacher-specialist from the respective grade. In Latvia, publishers choose two reviewers from the list published by the national authority.

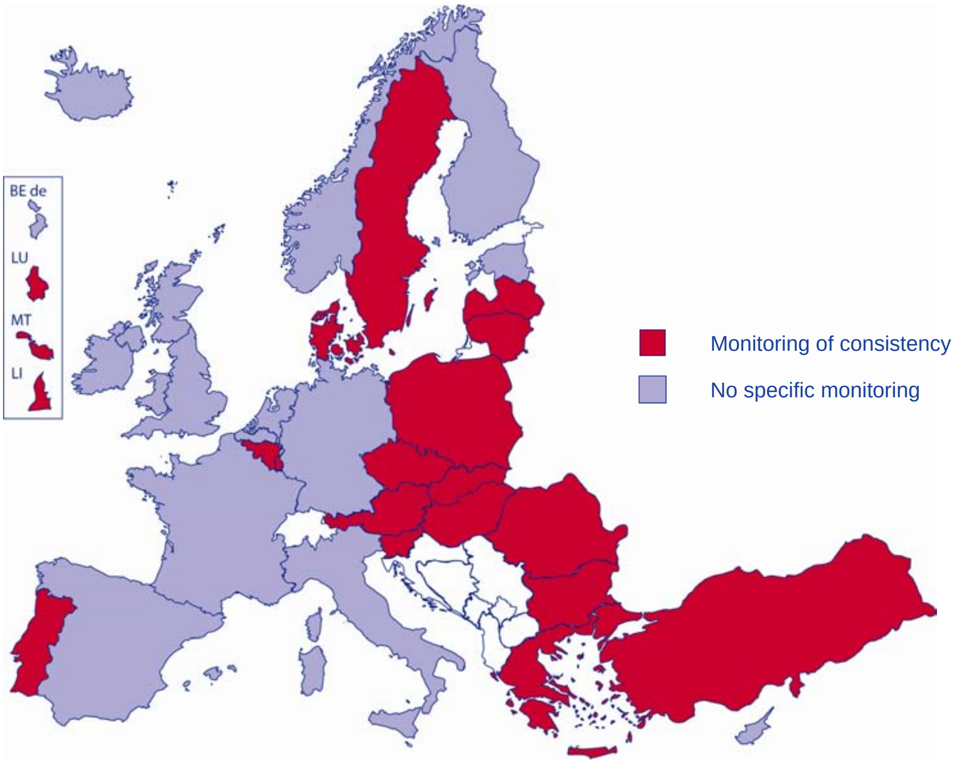
Several countries (Czech Republic, Hungary, Lithuania, Poland, Romania and Slovenia) mention that national institutes monitor the consistency between the content of textbooks and curricula. Compliance with the curriculum or other steering documents is often a condition of approval by the national authority before a textbook is included on the recommended list. In countries where schools have full autonomy for choosing textbooks, quality and compliance with the curriculum is driven by market forces. As pointed out by the United Kingdom (England, Wales and Northern Ireland), when a country

([8]) http://www.emu.dk/generelt/omemu/aboutemu.html
([9]) http://www.educacion.gob.es/ifiie/publicaciones.html

has a free market for textbooks and these are produced commercially, publishers have to strive for consistency and quality otherwise schools would not choose to buy their products.

In some countries (Belgium (French Community), Slovakia, Sweden, and Turkey), the preliminary consistency check between curricula and textbooks is reinforced by on-going evaluations and checks undertaken by school inspectors.

Figure 1.11: Monitoring the consistency between textbooks and the mathematics curriculum, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

Summary

In order to ensure that mathematics teaching continues to meet the changing needs of modern society, European countries set down regulations and recommendations in a variety of steering documents that differ in their degree of prescription and detail. However, the curriculum, or more generally, a central document defining the objectives, learning outcomes and/or content for mathematics, must be adhered to in the vast majority of European countries. Nevertheless, after taking into account the centrally defined curriculum framework, schools often have a large degree of autonomy to organise teaching and learning in ways that meet the needs of their students and/or local circumstances.

The most common way to disseminate the curriculum and other steering documents for mathematics teaching is through dedicated websites. In addition, many countries distribute printed copies of the curriculum to each school.

In all European countries, the mathematics curriculum has been revised in the last decade, often to incorporate a learning outcomes approach and/or the concept of key competences. The revisions often aim to improve the way mathematics is taught in classrooms and make it more relevant to students' everyday experiences. In many countries, the changes have reduced the focus on specific content areas and provided a more systematic approach to mathematics teaching. As a result of recent revisions, both learning objectives and learning outcomes are now usually prescribed in steering documents. In addition, mathematics assessment criteria are prescribed in two thirds of European countries.

The recommended taught time for mathematics instruction usually varies between 15 % and 20 % of the total taught time in primary education and is therefore the second most important subject after the language of instruction. In compulsory general secondary education, the share of time earmarked for the language of instruction and mathematics is lower than at primary level.

In many education systems, the effectiveness of the curriculum is evaluated using the results of national student assessment and information from schools' self-evaluation procedures. External evaluation of schools is performed in almost two thirds of European education systems.

Textbooks and learning materials are rarely prescribed by central education authorities. Instead, authorities usually make recommendations and monitor the consistency between mathematics textbooks and mathematics steering documents.

CHAPTER 2: TEACHING APPROACHES, METHODS AND CLASSROOM ORGANISATION

Introduction

The approaches and methods used for teaching mathematics in schools can have a huge impact on how much students learn in the classroom as well as on the quality of the learning that takes place. Appropriate teaching methods can improve students' level of understanding and help them master mathematical rules and procedures. The methods used also influence how students engage with and enjoy their learning, which, in turn, also impacts indirectly on how much and how well they learn.

Teaching methods underpin all learning in the classroom. They apply to subject content and how it is being taught, e.g. focusing on mathematical principles and processes or focusing on the application of mathematics in the real world. They also determine the nature of the interactions which take place in the classroom, such as those between the teacher and the whole class group, between the teacher and individual students, or between small groups of students.

This chapter provides an overview of educational research and policy developments in mathematics teaching and in classroom organisation. It summarises the teaching approaches and methods that are prescribed, recommended or supported in different European countries and places this information in the context of findings from international surveys which provide data on actual practice in schools.

2.1. Range of teaching methods: guidelines and practices

A number of research studies have investigated the most effective methods for use in teaching mathematics. The National Centre for Excellence in the Teaching of Mathematics (NCETM) in England conducted a one-year research study, *Mathematics Matters*, to identify the features of effective mathematics teaching (Swan et al., 2008). They concluded that it is not possible to identify a single best method, but found that there are many different types of learning and many different methods that should be applied, 'appropriate to the learner and the particular learning outcome required' (Ibid., p. 2). The project aimed to reach agreement on the types of learning that are most valued and the methods that are most effective in achieving these types of learning. The participants in the research concluded that the following types of learning are of value:

- fluency in recalling facts and performing skills;
- conceptual understanding and interpretations for representations;
- strategies for investigation and problem solving;
- appreciation of the power of mathematics in society.

They went on to agree that different methods are appropriate in developing these different types of learning, including, as an example, the use of higher order questions, encouraging reasoning rather than 'answer getting', and developing mathematical language through communicative activities (Swan et al. 2008, p. 4).

Similar to the findings of the NCETM study, Hiebert and Grouws (2009) after reviewing existing literature concluded that 'particular methods are not, in general, effective or ineffective. All teaching methods are 'effective for something' (p. 10). The authors found that different teaching approaches work for developing conceptual understanding in maths and for developing 'skill efficiency'. More precisely, the two important features of teaching when developing conceptual understanding are:

- 'discussions around mathematics including examining relationships between different areas of maths, exploring why different procedures work as they do and examining differences between different approaches; and
- requiring students to work on complex, open problems in mathematics'.

On the other hand, when developing skill efficiency, the review found that clear and fast paced presentation and modelling by the teacher, followed by practice by the students, worked well. However, they also found that this is not a simple dichotomy and it is not true that one approach works in one area only. They conclude that 'a weighted balance between the two teaching approaches might be appropriate, with a heavier emphasis on the features related to conceptual understanding' (Hiebert and Grouws 2009, p. 11).

Slavin (2009) investigated the quantitative evidence from a number of different studies with a view to evaluating competing claims about the effects of different mathematics teaching programmes. The development of teaching methods which involve students in co-operative learning have the most impact but professional development that improves classroom management and motivation also has benefits.

Hattie (2009) found in his extensive meta-analysis that the use of feedback can make a real difference in the mathematics classroom. The biggest difference was found when feedback included data or recommendations for students, then peer-assisted learning, explicit teacher-led instruction, direct instruction and concrete feedback to parents. Interestingly, he also found that the use of real-world applications of mathematics has a very slightly negative impact.

Kyriacou and Issitt (2008) reviewed 15 papers and concluded that 'the quality of teacher-initiated teacher-pupil dialogue to promote pupils' conceptual understanding needs to be improved' (p. 1). In particular, they found that the enhancement of students' understanding of how to make use of teacher-student dialogue as a learning experience was of particular importance when developing conceptual understanding.

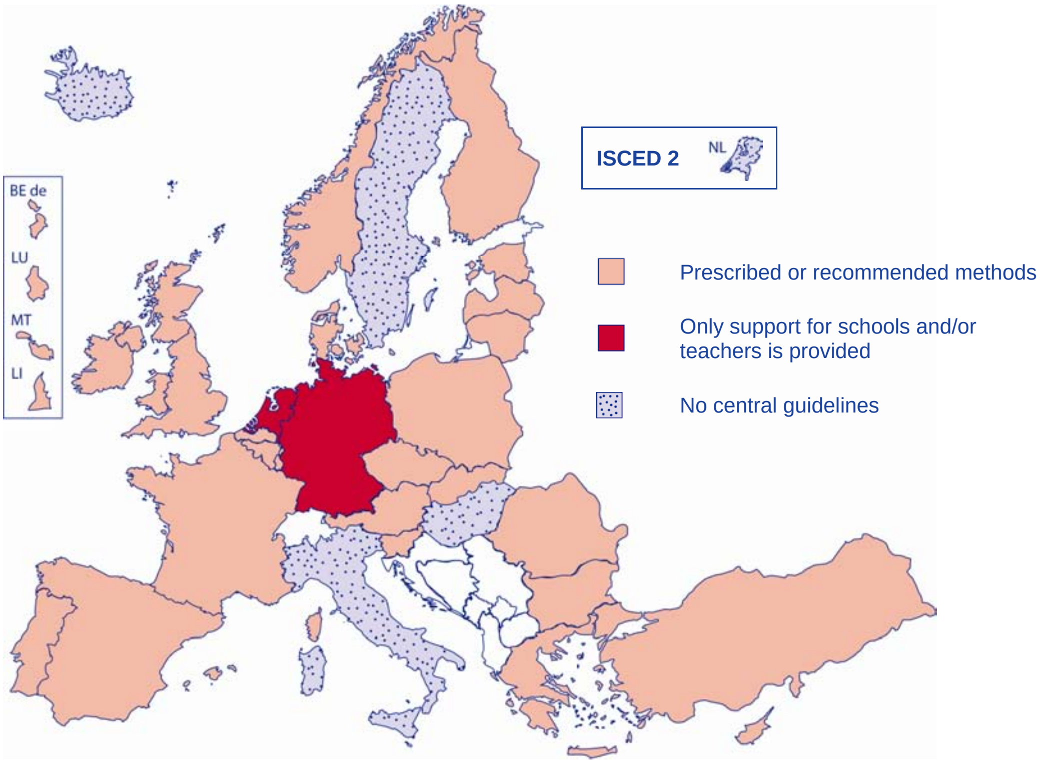
The research relating to different approaches and methods suggests that there is no one correct way of teaching mathematics, with some researchers arguing that different methods work in different contexts, and others that teachers ought to select the most appropriate method for their context and for a particular learning outcome, and that there may be complex relationships between what works. The conclusion would seem to be that professional development for teachers in a range of different methods, and allowing them to make decisions about what can be applied, when and why, is the best approach for improving teaching.

At policy level, central education authorities have some influence on the use of particular teaching methods. Across Europe, teaching methods are centrally prescribed or recommended in the majority of countries (see Figure 2.1). In contrast, in Germany and the Netherlands (ISCED 1), teachers or schools are only provided with central support in the form of web-based and other resources; and in five countries (Italy, Hungary, the Netherlands (ISCED 2), Sweden and Iceland), teachers do not receive any guidelines and it is up to them to choose which methods to use.

In **Hungary,** several approaches are noted in the National Core Curriculum as principles/aims of teaching/learning but no regulation or recommendation is provided on how to implement these principles in day-to-day teaching; this is a matter for individual teachers.

In **Sweden**, the 'Syllabus for compulsory education' describes the targets for pupils, and thus it has an influence on how teachers organise their teaching, but the methods, materials and tools are left to the discretion of teachers or teaching teams ([1]).

Figure 2.1: Central level guidelines regarding teaching methods in mathematics, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

In most countries, a variety of teaching methods is used. As a consistent research finding is that a wide variety of activities and methods can add value, this would appear to be a logical approach.

In **Greece,** for instance, the curricula and teaching manuals allow teachers to choose from various methods which, depending on the circumstances, can be used exclusively or in combination with others. In this framework, recommended teaching strategies include active learning via exploration/discovery, visits to a variety of locations including to those in the natural environment as well as to social/cultural destinations, presentations using appropriate teaching aids, dialogues between teacher and pupils or group conversations, direct instruction (using narration) and group cooperative instruction.

Another example of the promotion of a comprehensive approach to mathematics teaching can be found in **Germany**, where federal institutions launched a programme called SINUS (*Steigerung der Effizienz des mathematisch-naturwissenschaftlichen Unterrichts* – Increasing the efficiency of maths and science teaching) ([2]). The aim of the programme, which is organised at state *(Land)* level, is to make the teaching of mathematics and sciences more effective. The programme is based on eleven modules from which schools and teachers can choose. These cover issues such as problem-based learning, learning from mistakes, interdisciplinary approaches,

([1]) The Swedish National Agency for Education, http://www.skolverket.se/sb/d/2386/a/16138/func/kursplan/id/3873/titleId/MA1010%20-%20Matematik (in Swedish).

([2]) http://sinus-transfer.uni-bayreuth.de/startseite.html and student cooperation. The intended outcome is an effective change in teaching methods, but for this to be achieved all those involved must accept the process of innovation and integrate it into their own teaching.

In **Ireland**, at primary level, problem-based learning, discussion and the connection of subject content to daily life are all features of what is considered effective mathematics teaching, according to the curriculum document for mathematics and the accompanying guidelines for teachers. At post-primary level, these teaching methods are promoted during workshops provided as part of the implementation of ProjectMaths and during inspections conducted by the Inspectorate of the Department of Education and Skills ([3]).

Relating mathematics to daily life

All countries report that 'applying mathematics in real life contexts' is one of the aims of their curricula and/or other steering documents (see Chapter 1, Figure 1.7).

For example, in **Spain**, there is an emphasis on using what is familiar to students as a reference to learning contexts. At lower secondary level the mathematical content is adjusted so that it engages students and helps prepare them for adult life.

Similarly, in **Ireland**, it is recommended that pupils be given opportunities to use concrete examples both in the development of their mathematical understanding and in the development of their problem-solving skills.

In Estonia, outdoor learning is used to give students an understanding of longitudinal units at primary school, and at secondary school, teachers are encouraged to draw upon architecture and visual arts in order to explore geometry and symmetry ([4]).

In **Poland**, a key recommendation of the core curriculum is that the connection between mathematics and everyday life is pointed out in particular mathematical issues (e.g., percentages, units of measurement, field calculation, etc.) ([5]).

In **Italy**, a teacher training programme has been developed which aims to exploit teaching mathematics from the perspective of daily life using a problem-based approach ([6]).

Recent international survey data provides some further information about the methods being used in classrooms in European countries (Mullis et al. 2008, pp. 284-286). TIMSS 2007 data reveal that, according to teachers, students were often asked to relate what is being learned in mathematics to their daily lives (60 % of fourth graders and 53 % of eighth graders were asked to relate mathematics to their daily life in more than half of their lessons) ([7]). In Latvia almost all fourth grade students (94 %) had teachers who reported this activity in at least half of their lessons (see Mullis et al. 2008, p. 286). However, the relationship of mathematics activities to daily life may be more apparent or obvious to teachers than to students. European eighth grade students were less likely than their teachers to perceive that teachers related their mathematics lessons to students' daily lives (39 % of students on average, compared with 53 % of their teachers reported this). This difference in perception may also indicate that teachers do not provide clear enough explanations of how mathematics relates to daily life.

([3]) http://projectmaths.ie/

([4]) http://www.oppekava.ee/images/e/e2/Ouesoppest_imbi_koppel.pdf

([5]) The core curriculum document is available at: http://www.reformaprogramowa.men.gov.pl/images/Podstawa_programowa/men_tom_6.pdf

([6]) More information on the national plan, M@t.abel, is available on the website: http://www.indire.it/db/docsrv/A_bandi/apprendimenti_base_matematica.pdf

([7]) Eurydice calculations. Here and elsewhere, the Eurydice-calculated EU average for TIMSS data refers only to the EU countries which participated in the survey. It is a weighted average where the contribution of a country is proportional to its size.

Problem-based learning (PBL)

Another approach that is commonly promoted across Europe is problem-based learning. It focuses on acquiring knowledge and skills by analysing and solving representative problems. Learning often occurs in small groups under the guidance of a teacher who acts as facilitator. New information is acquired through self-directed learning and the problems encountered are used as a means to gain the required knowledge (Dochy et al., 2003).

Education authorities in a number of European countries recommend problem-based learning, or exploratory or investigative learning.

> At primary level in **Spain**, 'problem-solving processes are one of the central themes of mathematical activity and they should be the source and principal support for mathematical learning throughout primary education' ([8]). The mathematics curriculum in compulsory secondary education also refers specifically to problem solving as a basic topic of the curriculum ([9]).

> In **Cyprus**, problem-solving, investigation and exploration as the basis for the learning of mathematics is one of the key tenets of the new National Curriculum.

The TIMSS survey investigated problem-based learning activities for eighth grade students. According to the findings, 'applying facts, concepts and procedures to solve routine problems' or 'deciding procedures for solving complex problems' were regular activities in European classrooms, confirming the reports from countries that this is the approach they support. The proportion of eighth grade students whose teachers reported asking them to apply facts, concepts and procedures in more than half of their lessons varied from 39 % in Norway to 81 % in Bulgaria. The proportion of students whose teachers reported asking them to decide on which procedures to use for solving complex problems ranged from about 25 % in the United Kingdom (Scotland) and Norway to over 60 % in Cyprus and Romania. In contrast, working on problems for which there is no obvious solution was a less common activity. On average in participating EU countries, teachers of 23 % of eighth grade pupils reported working on problems for which there was no immediately obvious solution in more than half of their mathematics lessons. This ranged from 10 % in Norway to almost 40 % in Italy and Turkey.

PISA 2003, analysing student abilities in mathematics, created a separate scale measuring student proficiency in problem solving. It tested students' abilities to 'understand a problem situation, identify relevant information or constraints, represent possible alternatives or solution paths, select a solution strategy, solve the problem, check or reflect on the solution, and communicate the solution and reasoning behind it' (OECD 2004a, p. 46). The highest average results (ca. 547-548 points) were achieved by students in Belgium (Flemish Community) ([10]) and Finland. At the other end of the scale, Greece (448 points) and Turkey (407 points) had the lowest results (Ibid., p. 145). On average in participating EU countries, 16 % of students were only able to work in highly structured and straightforward settings, where they could deal with information available from direct observation or from very simple inferences (scored below Level 1). They were generally unable to analyse situations or solve problems that called for anything other than the direct collection of information, and are therefore characterised as weak or emergent problem solvers. Only 18 % of students on average in the EU reached the highest level of problem solving, and were able to construct their own representations of problems from pieces of information and then, in systematic ways, solve the

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([8]) Royal Decree 1513/2006, on national core curriculum for Primary Education http://www.boe.es/boe/dias/2006/12/08/pdfs/A43053-43102.pdf

([9]) Royal Decree 1631/2006, on national core curriculum for Lower Secondary Education, http://www.boe.es/boe/dias/2007/01/05/pdfs/A00677-00773.pdf

([10]) Here and elsewhere, EU average and Belgium (Flemish Community) results are Eurydice calculations.

problems and communicate their findings to others. The highest proportions of students able to solve problems at this level were found in Belgium (Flemish Community) (36 %), Finland (30 %), and Liechtenstein (27 %) (Ibid., p. 144).

Active learning and critical thinking

Moving away from the traditional teacher-dominated way of learning, active learning approaches encourage pupils to participate in their own learning through discussions, project work, practical exercises and other ways that help them reflect upon and explain their mathematics learning (Barnes, 1989; Forman, 1989; Kyriacou, 1992). Critical thinking is often linked to the ability to analyse, synthesise and evaluate information that is gathered through observation, experience or reasoning (Bloom et al., 1974; Scriven and Paul; 1987). It is used to solve problems, to choose between alternatives, and make judgements (Beyer 1995).

Almost all curricula and/or other steering documents refer to 'communicating about mathematics' as one of the competences that pupils need to develop (see Chapter 1, Figure 1.6) and cite active learning and critical thinking as good practice.

In **Belgium (Flemish and French Communities)** active learning is considered important in developing pupils' self-confidence, autonomy and creativity. Teachers allow time for reflection, which makes pupils more critical and encourages them to think more systematically and flexibly. The latter is advocated as good practice with regards to mathematics teaching.

In the **Czech Republic**, the Creative School *(Tvořivá škola)* project brings together 740 basic schools to exchange good practice on active learning, organise teacher training courses, prepare teaching materials and launch pilot classes in active learning. The *Reading and Writing for Critical Thinking* programme *(Čtením a psaním ke kritickému myšlení)* is an example of a programme promoting concrete, practical methods, techniques and teaching strategies ([11]).

Slovenia cites a model of developing physical/motor abilities alongside cognitive ability as an example of good practice. Students collect data from activities in 'sports education' and discuss the data from the perspective of the 'measuring domain'. Solving a problem is supplemented by an activity that helps to provide a rationale for the procedure, analyse solutions, encourage written and oral expression, and create models.

In **Spain**, activities such as reflection, establishing a working plan, adapting it, generating a hypothesis and verifying the validity of the solution are included in the core part of the curriculum.

The **United Kingdom** specifically mentions student self-evaluation as one of their strategies; this may also relate to the critical thinking and active learning approaches mentioned above.

PISA 2003 gathered information on similar learning methods, which they call *control strategies*. Several questions were intended to determine how well students control their own learning, set clear goals for themselves and monitor their own progress in reaching them. Among the European countries, control strategies were most often used in Germany and Austria, and the least in Finland and Sweden ([12]). However, the use of control strategies was not associated with improved mathematics performance in the majority of countries, although there were weak positive effects in Spain, Portugal and Turkey, and weak negative effects in seven European countries (Belgium, Denmark, Latvia, Hungary, the Netherlands, Slovakia and Sweden) (OECD, 2010).

([11]) http://www.kritickemysleni.cz/klisty.php?co=26/matematika

([12]) Eurydice calculations.

Memorising

In comparison with other methods, memorising is prescribed or recommended less frequently, but is nevertheless widely practiced as demonstrated by the findings of the TIMSS survey.

TIMSS 2007 data showed that teachers did often ask students to memorise formulas and procedures. Still, there were some differences between countries. At the fourth grade, frequent use of memorising strategies was reported for fewer than 10 % of pupils in four European countries: the Czech Republic, Germany, Sweden and Norway. Memorising formulas was more often reported in Latvia, Lithuania and Italy – ca. 45-65 % of students at the fourth grade had teachers who reported that this activity took place in half, or more than half of lessons (see Mullis et al. 2008, p. 286). Memorising formulas and procedures was more common at the eighth grade (on EU average, 24 % of students had teachers who reported this strategy in the fourth grade compared with 33 % in the eighth grade). According to teacher reports, memorising strategies were used in more than half of lessons for 60 % or more students in Bulgaria, Cyprus, Lithuania, Romania and Turkey at the eighth grade.

According to PISA 2003, 15-year-old students reported rather extensive use of memorising strategies, the most common of which was going through examples and remembering steps in procedures (OECD 2010, pp. 43-45). There were wide differences across countries in the extent to which memorising strategies were used. Students reported a comparatively higher use of these strategies in Greece, Hungary, Poland and the United Kingdom (Scotland). In contrast, in Belgium, Denmark, Finland and Liechtenstein students reported a comparatively low use of memorising strategies ([13]). Further analysis indicated that negative effects were observed between the use of memorising strategies and student achievement in mathematics (OECD 2010, p. 99). This suggests that either memorising is an ineffective strategy for learning mathematics, or that weaker students have a greater tendency to use this strategy.

Overall, there appears to be a great deal of variety across Europe in the approaches taken in whether particular methods are centrally controlled, or not, in whether the methods are published in some form, and in how they are subsequently implemented in schools. These differences in approach may be in part due to the lack of conclusive research findings in favour of one particular approach over another.

2.2. Classroom organisation: grouping of pupils

Much research has been conducted into the impact of grouping by ability in general, and in mathematics lessons in particular. Grouping can be used at whole-class level, with pupils being put into different ability streams for all their lessons, or with pupils being put into ability groups for different subjects; grouping can also take place within classes. Research has looked at the impact of grouping by ability on attainment as well as on attitudes and equity.

Sukhnandan and Lee (1998) conducted a systematic review of the existing research into the effects of 'streaming, setting and grouping by ability'. They judged that the findings were inconclusive, due to methodological limitations in the research and the difficulties of disentangling the effects from a wide range of other variables such as 'teaching methods, curriculum content, teacher and pupil expectations, resources, levels of ability and social characteristics' (p. 12). From the evidence of over 300 studies of tracking (grouping by ability at the whole-class level), Hattie (2009) concluded that the average effect size on attainment is small, and this applies in mathematics as well as other subjects. Hattie goes on to say that 'tracking has minimal effects on learning outcomes and profound negative

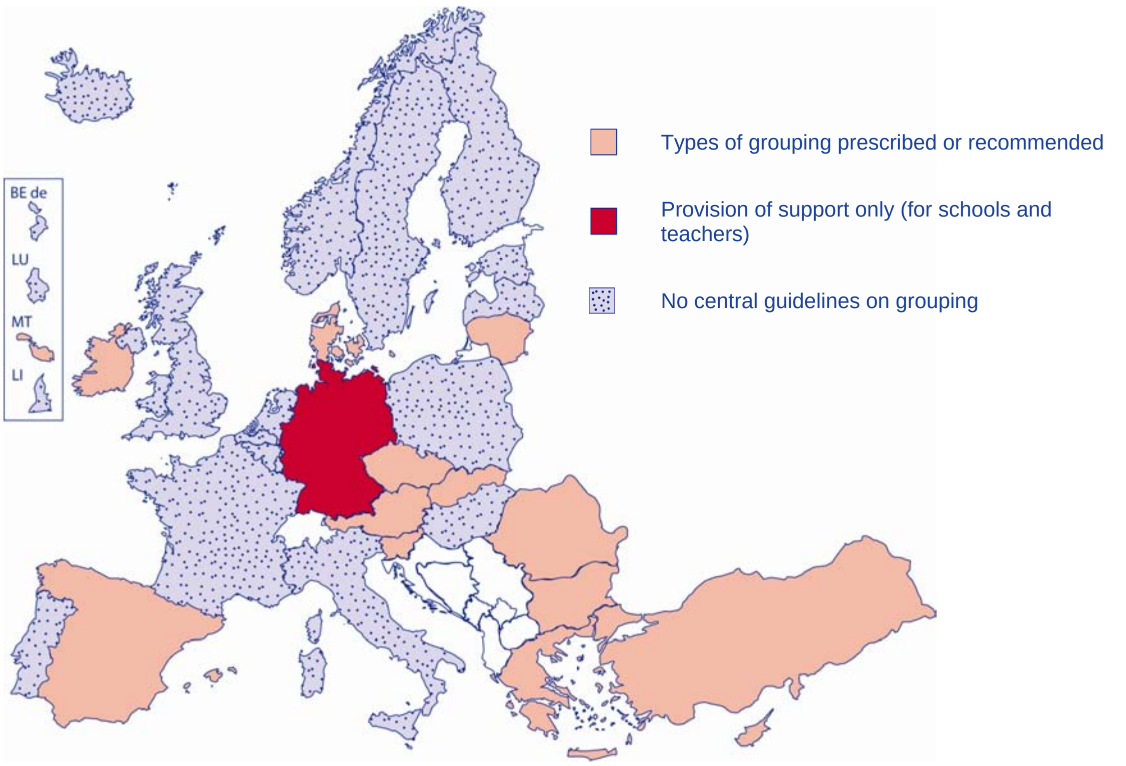
([13]) Eurydice calculations.

equity effects' (p. 90). He concludes by stating that 'the quality of the teaching and the nature of the student interactions are the key issues, rather than the compositional structure of the classes' (p. 91).

Kyriacou and Goulding (2006) reviewed studies that investigated the effects of grouping by ability and by gender in the mathematics classroom. They found that there were no clear and consistent findings in terms of the impact on motivation, although it does appear that a group of students who know that they cannot access higher grades in examinations will find it difficult to maintain motivational effort. They also found that the use of boys-only classes in co-educational settings does not have the intended consequence of reducing 'laddish' behaviour. More recently, Nunes et al. (2009) found that ability grouping in primary schools has a small positive impact on the mathematical reasoning of the top ability group, but hinders the progress of children in other groups.

Across Europe, education authorities take different approaches when it comes to prescribing or recommending the types of classroom organisation that their teachers use.

Figure 2.2: Central level guidelines on student grouping, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

As Figure 2.2 shows, less than half of European countries make recommendations or regulations on the grouping of students in schools. This may be done through the national curriculum or other steering documents. In some countries, such as the Czech Republic, general recommendations or regulations apply to different subjects, including mathematics.

In the remaining countries grouping arrangements are at the discretion of the school or individual teachers. However, France refers to certain procedural conditions in implementing group work at lower secondary level. Grouping is only permissible when mathematics teachers have submitted a scheme of work to the head of the school and the school's administrative council has approved the corresponding allocation of teaching hours.

Further information on the nature of grouping was provided by a number of countries from each of the categories, i.e. those with or without national guidelines on the issue. The data below refers to both national policies and actual practices, with the latter being especially informative with respect to the countries without national regulations or recommendations. The information on the types of grouping indicates that although a variety of methods exist, the most common approach is the grouping of pupils according to ability (see also Chapter 4). Ability grouping within classes or between classes is practiced in Belgium (Flemish Community), the Czech Republic, Spain, Lithuania, Malta, the Netherlands, Austria, Poland, Romania, Slovenia, the United Kingdom and Norway. In the majority of these countries, the two approaches are used simultaneously, although it appears that in primary education ability grouping between classes is less common.

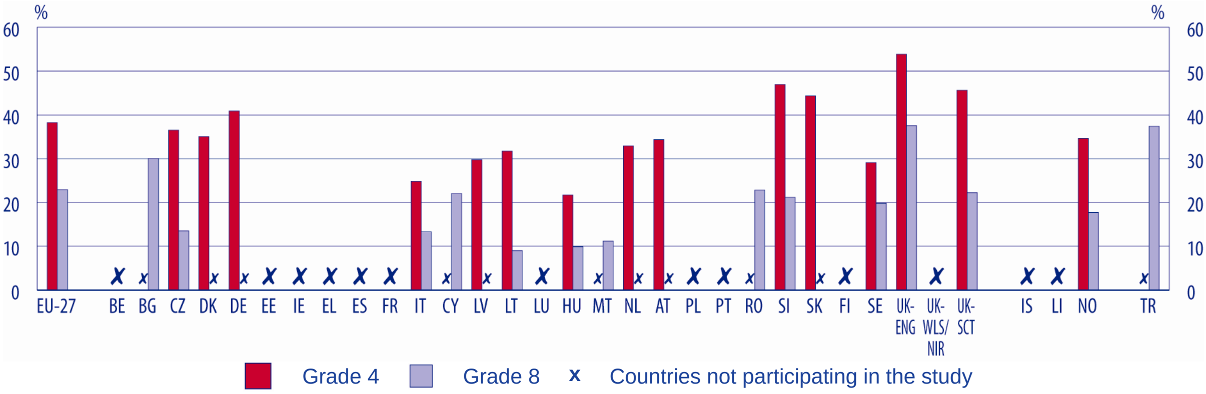
> In **Slovenia**, for instance, in years four to seven the school may deliver 25 % of lessons in ability groups; in years eight to nine schools may group pupils of the same year in ability groups or split them into smaller heterogeneous groups; alternatively, lessons may be delivered by two teachers simultaneously, or they may use a combination of all options.

Small group work and/or individual work in normal classes are also widely used methods. Belgium (German-speaking Community) advocates autonomous learning, where pupils are encouraged to work on activities at their own pace, although lessons are still taught to the whole class and work in small groups is highly recommended. In a similar vein, in Denmark, one recommended approach that encourages groups to gain a sense of autonomy is to split the class into four groups where each group works on a different activity.

TIMSS 2007 gathered data on the frequency of individual work and the most widely used grouping practices. Students reported how frequently they worked on problems on their own in class and how often they worked in groups. Individual work was widespread both at the fourth and eighth grades. On average, in participating EU countries, 78 % of fourth graders and 70 % of eighth graders indicated that they worked on problems on their own in at least half of their mathematics lessons. For each individual European country, at each grade, the percentage was at least 50 per cent (Mullis et al. 2008, p. 284). The highest percentages of students working on their own at least this often at the fourth grade were in Germany, Latvia and Austria (more than 85 %) and, at the eighth grade, in the Czech Republic and Sweden (more than 80 % of students).

The TIMSS 2007 report does not include data on how frequently students worked together in small groups. However, Eurydice calculations show that working in small groups was less frequent than individual work in European countries (see Figure 2.3). Moreover, collaborative working methods seemed to be less common at the eighth grade than at the fourth grade. On average in the EU, 38 % of fourth grade students reported working with other students in small groups in half, or more than half of their mathematics lessons. The percentages varied from 22 % in Hungary to 54 % in the United Kingdom (England). At the eighth grade, on average only 23 % of students indicated working together in small groups in half, or more than half of their lessons. In Bulgaria, the United Kingdom (England) and Turkey, group work was slightly more common – more than 30 % of eight graders reported often working in small groups. In contrast, in the Czech Republic, Italy, Lithuania, Hungary and Malta, fewer than 15 % of students at the eighth grade were working in small groups in half, or more than half of their mathematics lessons.

Figure 2.3: Percentage of fourth and eighth grade students who reported working with other students in small groups in about half of the lessons or more, 2007



	EU-27	BG	CZ	DK	DE	IT	CY	LV	LT	HU	MT	NL	AT	RO	SI	SK	SE	UK-ENG	UK-SCT	NO	TR
Grade 4	38.3	0.0	36.6	35.1	40.9	24.7	0.0	29.8	31.8	21.7	x	33.0	34.4	x	47.0	44.4	29.1	53.9	45.6	34.7	x
Grade 8	22.9	30.1	13.5	x	x	13.3	22.0	x	9.0	9.9	11.2	x	x	22.8	21.1	x	19.8	37.6	22.2	17.7	37.4

Source: IEA, TIMSS 2007 database.

2.3. Use of ICT and calculators in the mathematics classroom

Use of ICT

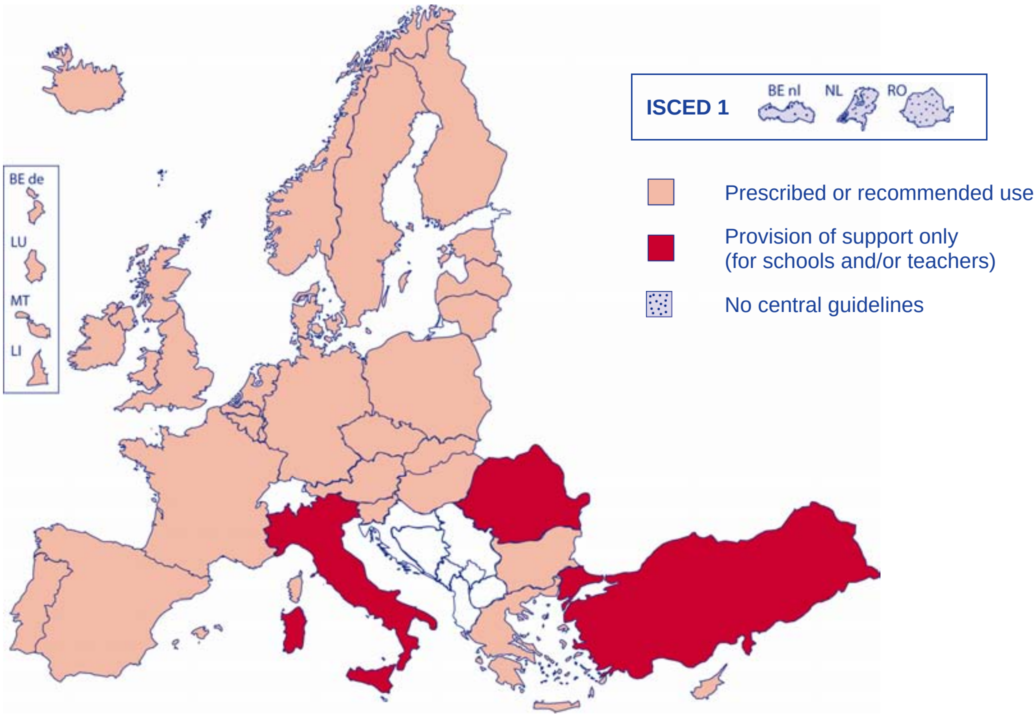
The research evidence into the use of ICT in the mathematics classroom has not found conclusive evidence about any definite benefits. Kyriacou and Goulding (2006) found that the use of ICT can have a positive effect in raising motivation, but it is important that the motivational effect is used in a way which enhances deeper understanding of mathematics. Slavin (2009) concluded that there is limited evidence that ICT does have a positive effect.

Many small studies, however, have found a positive impact from specific interventions using ICT. Burrill (2002) synthesised the findings from 43 studies and found that, with the right supportive classroom environment, handheld graphing devices can help students develop a better understanding of mathematical concepts, improve performance in assessments and improve problem solving skills. Clark-Wilson (2008) evaluated the use of TI-Nspire™ software and found that it could support the development of students' mathematical understanding. Roschelle et al. (2010) presented the findings from three studies on the use of technology in middle school mathematics in the United States. The studies 'evaluated the SimCalc approach, which integrates an interactive representational technology, paper curriculum, and teacher professional development' and found large positive impacts on student learning of more advanced mathematics.

As with the above research findings on teaching methods, it is not possible to say that ICT works to improve mathematics attainment per se, it is more likely that it works for certain things and in certain contexts. Research findings about effective pedagogy suggest that a variety of methods should make up a teacher's repertoire and it is likely that ICT should be one aspect of this repertoire. Effective teachers should know how and when to use it to its best advantage.

With respect to teachers' views and practices, the European Schoolnet 'ICT Impact Report' (2006) found that, although teachers recognise the value of ICT in education, they experience problems with the processes of adopting these technologies. As a result, only a minority of teachers has so far embedded ICT into teaching. Among the barriers to the use of ICT in teaching, the report mentions teachers' lack of ICT skills, poor motivation and confidence in using ICT, inappropriate teacher training, the absence or poor quality of ICT infrastructure and issues related to traditional schooling systems, amongst others. The report concludes that, in order to ensure realistic and holistic policy solutions, all these factors that prevent teachers from making full use of ICT must be identified and understood.

Figure 2.4: Central level guidelines on the use of ICT in the teaching of mathematics, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

As shown in Figure 2.4, the use of ICT for teaching mathematics is prescribed or recommended in all countries. This may range from very specific instructions to more general guidelines. For example, in Cyprus, it is recommended that applets are used for different content areas in mathematics, and that ICT is used for geometry investigations, statistical reasoning, and data collection. In Malta, pupils at secondary level should use spreadsheets, computer software relating to algebra systems, programming languages, and dynamic geometry. In Slovenia, the use of various ICT tools is recommended for developing mathematical concepts, research and modelling, practicing procedural routines, presenting results, and for assessment. In Portugal, the use of ICT is suggested for all subjects and at all educational levels ([14]), including in maths, and digital resources are provided to support teachers' work via the 'School Portal' ([15]). Portugal has also launched a programme called 'ICT Skills', which is a system for the professional development of ICT skills for all teachers. In Sweden, the use of ICT is an objective for students; they should develop their 'ability to exploit the possibilities offered by calculators and computers'. However, there are no regulations relating to specific ICT-related teaching methods ([16]).

([14]) http://www.metasdeaprendizagem.min-edu.pt

([15]) Portal das Escolas: https://www.portaldasescolas.pt/portal/server.pt/community/00_inicio/239

([16]) The Swedish National Agency for Education, http://www.skolverket.se/sb/d/2386/a/16138/func/kursplan/id/3873/titleId/MA1010%20-%20Matematik (in Swedish).

A small number of countries report e-learning as an example of good practice. In the Czech Republic, e-learning is promoted through the project 'Talnet' ([17]) as a new method for the 'study hour' for gifted students. In Italy, an e-learning programme 'SOS Studenti' provides an online learning environment especially designed for helping low achievers. In Poland the use of electronic versions of mathematics handbooks has been supported by the Ministry for some years now. In Liechtenstein, there are free online training tools for students and teachers ([18]).

The international survey data provides useful detail about how widely computers are available and how often they are used. According to TIMSS data, on average 57 % of fourth grade students and 46 % of students in the eighth grade have access to computers during their mathematics lessons. However, this availability is not equally distributed between different countries and it ranges from almost 95 % in Denmark at the fourth grade down to about 10 % in Cyprus at the eighth grade (Mullis et al., 2008).

The number of available computers in different European countries is very varied, as is the degree of detail provided in regulations and recommendations as to how they should be used.

> In **Estonia**, the National Curriculum for Basic Schools sets out specific outcomes for ICT use: at the first study stage (grades 1-3), students should learn to use digital learning objects (worksheets, learning programs, etc.); at the second study stage (grades 4-6), students should be able to use ICT for numerical calculations, and for checking calculations which are made on paper. In addition, at the second stage, students should be able to employ suitable study skills, and find necessary help and appropriate data resources from several information sources.
>
> In **Latvia**, the curriculum also sets out specific outcomes in terms of ICT use: at primary level, students should know how to use computers to obtain information; at the completion of secondary education, students should know how to use calculators/computers to process information. However teachers retain autonomy with regard to how, and to what extent to use these information technologies.
>
> In **Spain**, technological media are seen as essential tools to teach, learn and practice mathematics, and it is thought that their everyday use in the workplace should be reflected in the classroom. There is a strand in the national curriculum that incorporates the use of ICT: 'Information processing and digital competence'. This strand seeks to provide students with number skills, such as comparison or approximation, and to introduce students to graphic and statistical language. At lower secondary level, students also use spreadsheets and this activity leads to 'question formulation, comprehension of ideas and report writing'. Dynamic geometry programmes are also used at this level, leading to the analysis of properties, the exploration of relationships and the formulation and validation of conjectures.

Four countries commented on guidelines for teachers' use of ICT in the classroom:

> In **Iceland**, teachers are encouraged to emphasise visual presentation by making use of video, calculators and computer programs in order to explain mathematical concepts and help students express their views visually. In **Italy** and **Spain**, LIMs (interactive whiteboards) have recently been promoted nationally and this has led to a national strategy being developed to support the use of ICT in daily teaching. In **France**, the use of computer software (for example, for dynamic geometry) is recommended at least for mathematics teachers if not also for students.

([17]) http://www.talnet.cz/talnet_new/ukazky-z-kurzu

([18]) Available at www.schultraining.li and www.lernareal.ch

TIMSS survey data provides further detail about how computers are being used. Even where availability was high, computer use appeared to be relatively infrequent in mathematics classes. For example, in Lithuania, where according to teachers 73 % of eighth grade students had access to a computer for learning mathematics, only 5 % of them were using computers for processing and analysing data in half, or more than half of their lessons (Mullis et al. 2008, p. 301). Overall, for all forms of use (discovering principles and concepts, practising skills and procedures, looking up ideas and information, and processing and analysing data), the figures reported were below 10 per cent in the fourth and eighth grades of almost all countries. The only exception was the Netherlands (30 %) and the United Kingdom (England 10 % and Scotland 20 %), where fourth grade teachers reported more frequent use of computers for practicing skills and procedures.

Data therefore suggest that although computers are available, they are not used widely in mathematics lessons. This is true both for countries where the national curriculum contains a statement about the use of computers in mathematics lessons, as well as for those countries which do not have any such prescription or recommendation. The 2011 Eurydice report on 'Key Data on Learning and Innovation through ICT at School in Europe' arrives at similar findings. It shows that teachers are encouraged through central level recommendations, suggestions or support material to use a variety of ICT hardware and software in the classroom – and this applies in almost all European countries to all core subjects of the curriculum, including mathematics. However, in terms of the actual use of ICT in the classroom, evidence shows that teachers make little use of these opportunities and so a large implementation gap currently remains.

Use of calculators

There is an ongoing debate about whether the use of calculators improves or hinders student achievement in mathematics. Most studies seem to conclude that calculators might be useful, but only for specific activities. Hattie (2009) found a low but positive effect on attainment from the use of calculators in mathematics. However, calculators were useful only in certain situations:

- when they were used for computation, drill and practice work, and for checking work;
- when they reduced the cognitive 'load' on students so they could attend to other, more mathematical, concepts; and
- when used for a pedagogical purpose in which they were to be an important element in the teaching and learning process.

Hembree and Dessart (1986) in their meta-analysis of 79 studies also found that the use of calculators, alongside traditional teaching methods, improved students' skills in mathematics exercises and in problem solving, at all but grade 4. The authors state that at grade 4 the sustained use of a calculator 'appears to hinder the development of basic skills in average students'. Similarly, Ellington (2003), in another meta-analysis of 54 studies, found that the use of calculators improved students' operational and problem solving skills when they were used in teaching and assessment, although not when they were used solely in teaching.

The curricula in almost all European countries, excluding Belgium (German-speaking Community) and Romania prescribe, recommend or provide support for the use of calculators in mathematics teaching. Some countries mention certain limitations.

> In **Liechtenstein**, it is recommended that in order to ensure the development of basic skills such as mental arithmetic and written arithmetic techniques, calculators should not be used until a student reaches secondary age. In **Ireland**, calculators may be used from the age of about 10, by which time the child should have acquired a

mastery of basic number facts and a facility in their use. In the **United Kingdom (Scotland)** and **Spain**, calculators have a place in learning and teaching when used in a problem solving way, but their use is not intended to replace basic skills development. In **Germany** and the **Netherlands**, guidelines on the use of calculators only refer to lower secondary level. In **Cyprus**, on the other hand, calculator use is recommended only for primary students.

Information about the use of calculators in assessment, as compared to the use in classrooms described here, is provided in Chapter 3.

Findings from TIMSS show that a slight majority (53 %) of the teachers of fourth grade students reported that calculators were *not allowed* to be used in the mathematics class. However, there were significant variations between countries. Countries where calculator use is largely restricted included Italy, Latvia, Hungary, Austria and Slovenia where ca. 85 % or more fourth grade students were *not allowed* to use calculators. In contrast, in Denmark, Sweden, the United Kingdom (England and Scotland) and Norway, ca. 85 % or more were *permitted* to use calculators (Mullis et al. 2008, p. 298). In general, even in those countries where calculators were widely allowed, teachers rarely reported using calculators often (i.e. in half, or more than half of lessons). The highest percentage reported for such frequent calculator use was in Denmark, where teachers reported that 23 % of students used a calculator in half, or more than half of their lessons for solving complex problems. In other European countries, the reported percentages were around 10 % or even lower.

The situation was very different at the eighth grade where a majority of students were allowed to use calculators, and used them quite often. At the eighth grade, on average in the participating EU countries, 87 % of students were permitted to use calculators, with a range of between 30 % (Cyprus) and 100 % (Malta and Sweden). On average in European countries, calculators were used in approximately half, or more than half of lessons to solve complex problems (43 %), do routine computations (33 %), and check answers (28 %).

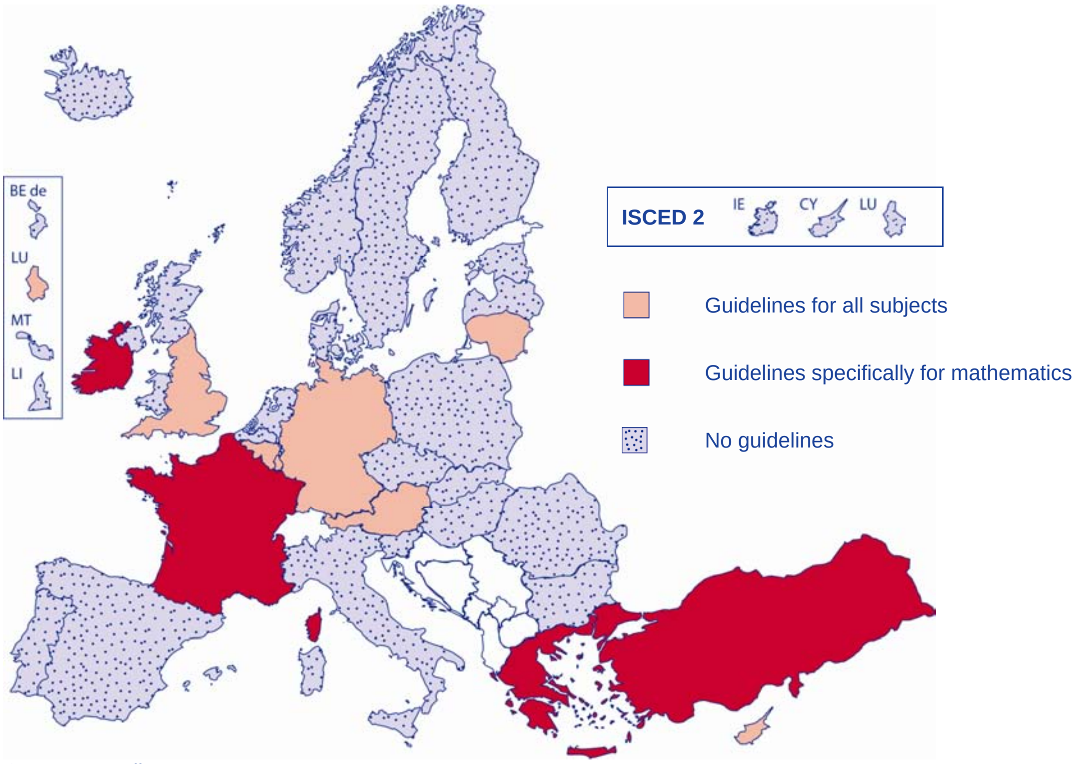
2.4. Assigning homework

A considerable number of studies have examined the relationship between achievement and homework. The facets investigated include the amount of homework assigned and actually completed, as well as the time spent on tasks (Marzano and Pickering, 2007).

Hattie (2009, p. 234) concludes that homework has an overall positive effect on learning 'but there are some important moderators'. He quotes studies by Cooper (1989) which demonstrate that effects are larger for students in later stages of education than earlier, and larger in some subjects than others, with the smallest effects being seen in mathematics. Cooper also found that the positive effects of homework are related to the length of the homework with, on the whole, shorter tasks being better. Similarly, Trautwein et al. (2002) concluded that the frequency of mathematics homework has a positive impact on achievement, whereas homework that requires longer periods of time to complete does not. The overall picture in the research with regards to homework is not straightforward. Hattie concludes that 'the effects are highest, whatever the subject, when homework involves rote learning, practice, or rehearsal of the subject matter' (p. 235).

In most countries, central education authorities do not provide guidelines in steering documents on mathematics homework policy for students in primary or lower secondary school (see Figure 2.5). Usually homework policy is left to the discretion of individual schools and teachers. Given the limited positive findings seen in the research between an emphasis on homework and attainment in mathematics, this may be considered a sensible approach. However, this would still leave teachers with scope to set large amounts of homework, so provision of guidance *limiting* the amount of homework set may be more valuable.

Figure 2.5: Central level guidelines on assigning mathematics homework, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

In most countries where guidelines exist, they are general guidelines which apply to all subjects. The exceptions are in Ireland (primary education), France (lower secondary level), Greece and Turkey where specific guidelines for mathematics lessons exist.

In **Ireland**, at primary level, homework is envisaged as a reinforcement exercise, offering an opportunity to widen experiences begun in the classroom, for example, when working on capacity, students may be asked to find the area of a room at home. Homework is considered to help students develop organisational skills and the ability to work independently. Homework is envisaged as a link between home and school. Curriculum documents also emphasise the importance of informing parents of the correct terminology and methods being used by the children in mathematics. In so doing, teachers are encouraged to make homework assignments realistic, practical and relevant. Teachers are also urged to set alternative forms of homework, for example, to carry out research in the local library or use measuring skills in cookery.

In **France**, at lower secondary level, mathematics homework is compulsory and teachers must collect and correct it regularly.

In **Greece**, official Ministry of Education documents note that homework must comply with and be complementary to the content of the school text book; it should not be intensive; and it must require minimum assistance from parents or any other person.

In **Turkey**, curriculum documents state that homework should be given depending on a student's motivation; performance homework (e.g. research projects) should be given to students to evaluate their ability in critical thinking, problem solving, understanding what they read, creativity and doing research; some homework should be suitable for peer evaluation; homework may be designed to contribute to portfolios.

There is some consensus that the purpose of homework should be to consolidate learning, and that it must be of an appropriate level for the student. Cyprus states that homework should be interesting, and not overly repetitive. In the French community of Belgium, the Ministerial Circular of 13 May 2002 regulates homework during primary education: it mentions that homework should be adapted to the level of competences and study rhythm of each pupil and that it should take 20-30 minutes to complete ([19]).

Homework policies are also often linked to the issue of parent involvement in the learning process. In the United Kingdom (Scotland), homework is viewed as a task that can help to reinforce the interaction between parent and child. Education authorities in Cyprus stipulate, however, that homework should be accomplished without parental support. In France, homework is forbidden for primary school students, however in practice, if strongly requested by parents, teachers do assign it.

A further important issue is the time spent on homework. Recent national reports in Romania revealed that one of the negative factors influencing student motivation in learning mathematics is too much time spent on homework. Indeed, compared to other countries, the amount of time spent on homework in Romania (see TIMSS results below) appeared to be one of the largest. Consequently, the regional and central authorities have made recommendations to restrict homework to 30-45 minutes, which still seems relatively long compared with other countries.

The TIMSS report (Mullis et al. 2008, pp. 302-307) contains data collected from teachers about their emphasis on mathematics homework. This is based on teachers' responses to two questions about how often they assign mathematics homework and how long they expect that homework to take. The Index of Teachers' Emphasis on Mathematics Homework (EMH) was calculated aggregating the questions into three categories. Students in the 'high homework' category had teachers who reported giving them relatively long homework assignments (more than 30 minutes) on a relatively frequent basis (in about half, or more than half of lessons). Conversely, students in the 'low homework' category had teachers who gave short assignments (less than 30 minutes) relatively infrequently (in about half the lessons or less often). The 'medium homework' category included all other possible combinations of responses.

At the fourth grade, on average in participating EU countries, homework was not widespread. Only 13 % of students had teachers who placed a high emphasis on mathematics homework, while 41 % of students had teachers who gave only short assignments and relatively infrequently or no homework at all. Emphasis on homework varied across countries. In Italy, the emphasis was the highest: 35 % of fourth grade students had teachers who reported giving relatively long homework assignments on a relatively frequent basis. In contrast, the Czech Republic, the Netherlands, Sweden, the United Kingdom (England and Scotland) had most students (more than 75 %) with teachers who placed little emphasis on mathematics homework. In the Netherlands and the United Kingdom (England), this low emphasis might be a reflection of national or local policies that restrict homework for this age group.

Eighth grade teachers placed more emphasis on mathematics homework. On average in the EU countries, teachers of more than one third of students (37 %) reported giving relatively long homework assignments on a relatively frequent basis. However, there was a wide range across countries. Exceptionally high percentages of students in Italy and Romania (70 %) had teachers who gave them a lot of homework. On the other hand, more than 50 % of students had teachers who gave short assignments relatively infrequently in the Czech Republic (77 %), Sweden (63 %) and the United Kingdom (England, 59 %, and Scotland, 55 %) (Mullis et al. 2008, p. 305).

([19]) http://www.gallilex.cfwb.be/document/pdf/21557_007.pdf

Findings from PISA 2003 showed that 15-year-olds in Europe typically received between 3.7 hours (Finland) and 10.5 hours (Italy) of homework a week, with a range for mathematics homework from 1.3 hours (Sweden) to 4.1 hours (Poland) per week (see OECD, 2003, Table A.5, p. 152).

The relationship between homework and achievement seems to depend on education level. TIMSS results show that, at the fourth grade, there is no relationship between the amount of homework and student achievement ([20]), while at the eighth grade a positive association was observed in several countries. This might be explained by the varying purposes of homework. For example, homework might be emphasised for higher achieving students in order to stretch and challenge them. However, homework might also be assigned to lower achieving students in order to provide them with further practice or consolidation. Thus, similar levels of homework can be associated with different levels of achievement, resulting in no straightforward overall relationship between levels of homework and achievement.

At the eighth grade, however, in participating EU countries, on average there was no overall relationship between the emphasis on homework and student achievement. The average scores of European students in each homework category were similar (492, 493 and 493 scale points respectively) and the correlation was non-significant ([21]). However, in the Czech Republic, Hungary, Malta, Romania, Slovenia and the United Kingdom (England and Scotland), a higher level of homework was associated with higher attainment. For example, in the United Kingdom (England), the 18 % of students whose teachers reported giving relatively long homework assignments on a relatively frequent basis scored on average 552 points in mathematics, the 23 % in the medium category scored an average of 520 points, and the 59 % whose teachers gave little homework on average scored 499 points (Mullis et al. 2008, p. 304).

Findings for the older students surveyed in PISA 2003 showed additional interesting patterns. The hours of *total* homework across all participating countries were positively associated with achievement (that is, the more homework that was assigned overall, the higher the students achieved in mathematics). Conversely, there was an overall negative association between hours of *mathematics* homework and achievement: the more mathematics homework was assigned, the less well students achieved in mathematics. Higher-performing students do more homework generally, but do less mathematics homework. The PISA report suggested that this might be linked to the nature of mathematics: that the most able students might learn their mathematics mainly in school or finish standard homework in less time, while less able students might struggle more and therefore need mathematics homework (OECD, 2010). Unfortunately, as PISA did not examine the nature of the homework, its supervision or monitoring, deeper explanations were not possible.

2.5. National surveys and reports to support evidence-based policy on mathematics teaching methods

Collecting, analysing and disseminating evidence on mathematics instruction is an important way of informing policy development and contributing to the improvement of classroom practices. It also indicates how far existing policies are being implemented and whether or not they are based on evidence of best practice.

([20]) Eurydice calculations. The correlation between the Index of Teachers' Emphasis on Mathematics Homework (EMH) and student achievement was very low and not significant in all participating European countries except Latvia (where no teachers assigned much homework).

([21]) Eurydice calculations: The correlation between the Index of Teachers' Emphasis on Mathematics Homework (EMH) and student achievement in mathematics.

Many European countries do not have any national organisations in place to routinely carry out such reporting activities. In others, these activities are undertaken by pedagogical centres or research institutes, which were either set up by the Ministries of Education or work in close collaboration with them. These institutions usually have the tasks of producing statistics, monitoring developments in the education system and analysing and interpreting trends. In their work, they often consider results from both national assessments and international surveys on student learning outcomes.

In **Austria**, the Federal Institute for Education Research, Innovation and Development of the School System (BIFIE) comprises several centres that advise on, and evaluate the implementation of curriculum reform, prepare test instruments, draft periodic reports on results of national educational research and design innovative pilot projects.

In **Sweden**, a national centre for mathematics education located at Göteborg University ([22]) undertakes inquiries for the Ministry of Education and Research and cooperates with other key national and international education partners and stakeholders. They undertake work on various aspects of mathematics teaching, including publishing texts for teacher education and professional development, organising conferences, developing support for municipalities and schools. They also provide a national reference library and a 'Maths-Lab' for hands-on activities.

In the **United Kingdom (Scotland)**, in addition to the statistical unit that oversees the collection of data from national tests in mathematics, there is the Scottish Qualifications Authority (SQA) which collects data on national qualifications for all subjects, including mathematics, and provides in-depth analysis after synthesizing the information. Learning and Teaching Scotland (LTS) is another government-supported body that collates research data relating to all areas of the curriculum, both national and international.

Several other countries – Belgium (French Community), Denmark, Germany and Finland – mainly rely on research and analyses provided by universities and other independent research associations.

The **Danish** School of Education (Aarhus University) is a postgraduate university school that conducts research in the area of education studies. In **Germany**, the 'Mathematicians' Union' ([23]) produces research, develops projects and organises conferences to disseminate evidence in the field of mathematics teaching and learning. In **Finland**, too, there is no official structure for collecting information on mathematics education, but there are many associations developing and sharing the latest research and data in this area.

Amongst other topics, these bodies also report on teachers' choice of teaching methods and activities for use in mathematic lessons. Roughly half of all European countries reported implementing and using such national surveys or reports.

A number of countries (Belgium (Flemish Community), Austria, Spain, Latvia, Malta, Norway, and the United Kingdom (Scotland)) report using surveys to investigate teachers' choice of methods and activities, with Malta and Norway both making specific mention of using the TIMSS surveys to gather such information. Norway has also used the SITES 2006 survey to inform educational development ([24]). In Spain, the publication of education indicators periodically provides data about the most frequently used teaching methods, as indicated by teachers in the questionnaires for the national assessments of primary and secondary education ([25]). In Belgium (Flemish Community), the surveys *(Periodieke Peilingen)* (see Chapter 4) include research on the link between teaching methods and differences in learning outcomes.

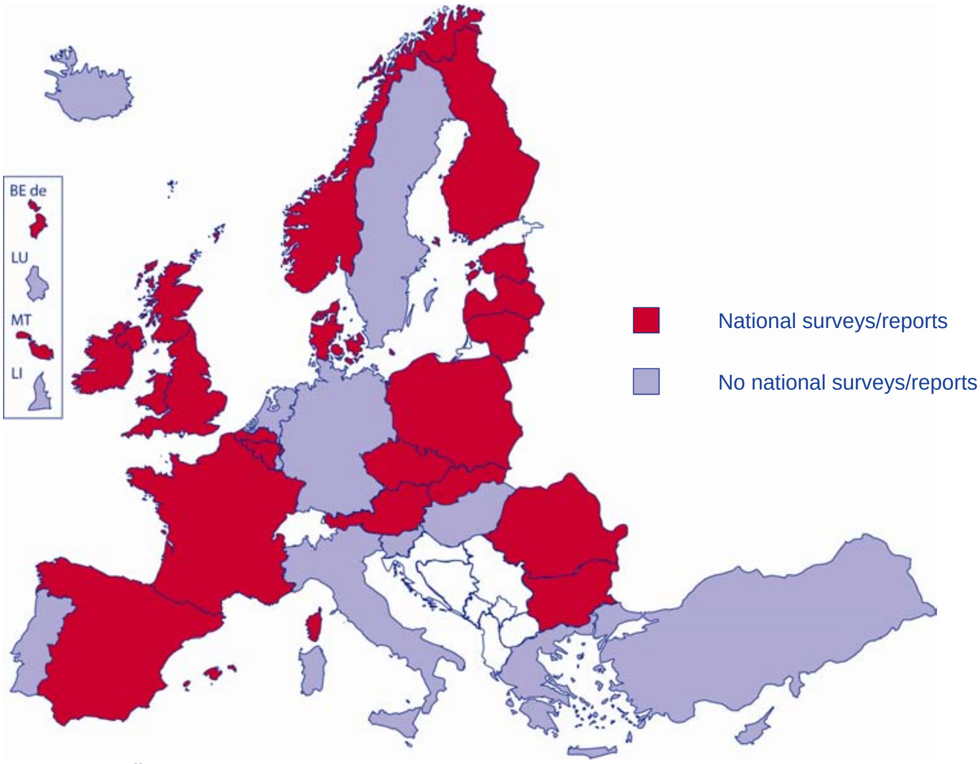
([22]) www.ncm.gu.se/english

([23]) https://www.dmv.mathematik.de/

([24]) http://www.sites2006.net/exponent/index.php?section=29

([25]) http://www.institutodeevaluacion.mec.es/dctm/ievaluacion/indicadores-educativos/ind2009.pdf?documentId=0901e72b80110e63

Figure 2.6: National surveys on teachers' choice of teaching methods and activities, 2010/11



Source: Eurydice.

Countries (Belgium (French Community), the Czech Republic, Bulgaria, France, Malta, Romania, Slovakia and the United Kingdom (England, Wales and Northern Ireland)) also use school inspections to investigate which teaching methods are being used. Often teaching methods are analysed and discussed and teachers are given advice during inspection visits. Information from inspection visits is subsequently shared via regional or national reports.

Some of the conclusions of these national surveys and reports indicate current weaknesses in the teaching of mathematics. The French Community of Belgium reports that inspections reveal some poor coverage of the curriculum ([26]). The report by the Danish Evaluation Institute suggested, among other things, that more work should be done to encourage non-mathematics teachers to use mathematics in their subjects. Finland reports that whole-group instruction is favoured over individual student work. Lithuania notes that there are a large number of students who are not actively involved in the learning process, whilst one of the main findings in Poland is that teachers set aside too little time for students to find their own strategies to solve problems and use mathematical models by themselves.

([26]) http://www.enseignement.be/index.php?page=24234

Summary

This review of the approaches and methods used in mathematics teaching across Europe reveals evidence of the degree of central control over practice in the majority of countries. Current regulations, recommendations or support are generally in line with research findings which indicate that there are no best approaches to teaching mathematics and that teachers need to choose appropriate methods and strategies to suit the topic, the type of student and the particular learning context. International survey data provide evidence that a range of approaches is being used in practice. However, in order for teachers to be able to provide this flexibility in pedagogy and be capable of selecting the most appropriate approach or method at any given time, it is crucial that they have access to effective professional development (see Chapter 6).

Despite the variety of teaching methods in use there is a clear evidence of emphasis on a number of particular methods. The use of problem-based learning, exploration and investigation is the focus in a number of countries, as is the use of real life contexts to make mathematics more relevant to the students' own experience. A method which was found to be common in both TIMSS and PISA, but less so in central level guidelines, is the use of memorising strategies.

There is less central involvement of the way mathematics classes are organised (e.g. streaming, setting or grouping), with two thirds of countries reporting some central guidelines. The most common form of grouping is by ability. The TIMSS data suggest that, having students work on their own is much more common than having students work in small groups. The findings show that, on average, 78 % of fourth grade students and 70 % of eighth grade students work on their own in more than half of lessons, compared to the 38 % and 23 % respectively who frequently work in small groups.

The use of ICT in the mathematics classroom is prescribed in the majority of countries. Research findings show that certain uses of ICT can have a positive benefit in certain contexts, which suggests that regulations should be detailed if there is to be a positive impact, or, as with the different teaching methods, teacher expertise in selecting the most appropriate use of ICT should be at a high level. As with the selection of the most appropriate teaching methods, this implies the need for extensive professional development. The TIMSS data show that access to ICT in countries in Europe is very variable – ranging from 22 % to 95 % of students at the fourth grade and 11 % to 81 % at the eighth grade. However, computers are rarely used in practice in mathematics lessons.

Research into the use of homework and findings from international surveys suggest that it can have limited positive benefits, especially with younger students and especially in mathematics when compared to other subjects. Many countries in Europe do not give central guidance on the use of homework, although some do give advice on appropriate time allocation. Based on other evidence, it may be more appropriate for restrictions to be placed on the amount and type of homework given since research suggests that it is most useful when used to practice skills.

Approximately half of all European countries described monitoring the use and success of different teaching methods on an on-going basis. This was done through a combination of assessment results and inspection procedures.

CHAPTER 3: ASSESSMENT IN MATHEMATICS

Introduction

Student assessment is an essential tool for monitoring and improving the teaching and learning process. Effective use of assessment for learning has been shown to be beneficial for all students, including those with lower achievement. Across Europe, student assessment takes a variety of forms and uses different assessment instruments and methods. The models used may be internal or external, formative or summative, and results can be used for different purposes (EACEA/Eurydice, 2009; OECD, 2011).

However, research shows that assessment is too often used for grading students, and less for helping them to improve their performance. Improving knowledge and skills requires a more extensive use of different forms of assessment that provide feedback and so make it possible to identify and address problems at an early stage (European Commission, 2008). Teachers play an important role in student assessment and they need training and guidance to deal effectively with these issues.

This chapter analyses national level guidelines and practices related to the use of different forms of assessment, including national tests. The chapter also looks at whether mathematics is included in school leaving examinations at the end of upper secondary education. The use of mathematics assessment data as well as national reports and surveys for improving the quality of instruction and supporting new policy developments is briefly discussed at the end of the chapter.

3.1. Improving learning through diverse and innovative forms of assessment

Before considering official guidelines on mathematics assessment in European countries, it is worthwhile looking at the general trends in mathematics assessment in schools as revealed by data from international surveys. Both TIMSS 2007 and PISA 2003 included some questions to teachers and school heads regarding common assessment practices.

TIMSS 2007 data (Mullis et al. 2008, pp. 309-310) shows that teachers of eighth grade students placed most emphasis on classroom tests as a way of monitoring students' progress in mathematics. Teachers used classroom tests to some extent for nearly all students. In participating EU countries on average, 64 % of students had teachers who reported putting the major emphasis on classroom tests and 32 % had teachers who reported putting some emphasis on them. Other commonly reported ways of monitoring student progress was teachers' own professional judgement. Fifty-six per cent of eighth grade students had teachers who placed major emphasis on their professional judgment and a further 40 % placed some emphasis on this.

TIMSS 2007 also asked how often mathematics teachers of eighth grade students set mathematics tests or examinations. The results showed that almost half (44%) of eighth grade students were given mathematics tests about once a month, on average, in participating EU countries. About one-third (32 %) were given mathematics tests or examinations every two weeks (or more frequently). However, this varied considerably by country. In the Czech Republic, almost all students (97 %) were given a test at least every two weeks. In Lithuania, Hungary and Romania teachers also often reported giving mathematics tests or examinations (for 70-75 %) students every two weeks or more. There were also several countries where the majority of students were given mathematics tests or examinations no more than a few times a year, including Slovenia, Sweden and the United Kingdom (England and Scotland).

Two main forms of assessment can be identified: those where results are used for formative purposes, that is to improve future teaching and learning, and those which are used for summative purposes, that is to provide evidence of student achievement over a certain study period.

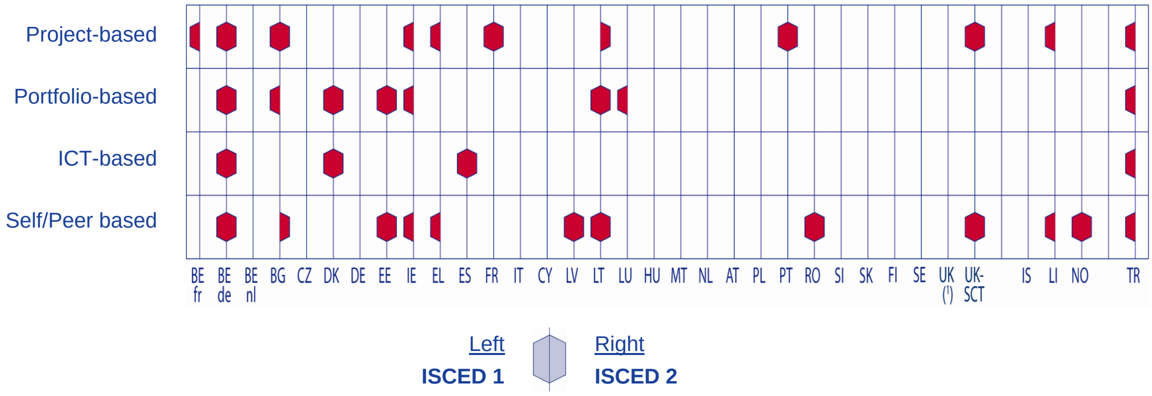
In 1998, Black and Wiliam published their highly influential report on formative assessment. They stated that assessments become formative when the information produced from them is used to adapt teaching and learning to meet student needs. The report synthesised findings from a large number of research projects and concluded that formative assessment clearly raises levels of attainment, but that its use could be improved in many cases. They went on to detail which strategies teachers should adopt in order to realise the improvements. This original report did not focus on any particular subject area, but in 2007 Wiliam went on to define what this could look like specifically in the mathematics classroom. As with the general review, this focused on ways of providing feedback to students, and also ways in which classroom practice can be adapted.

More recently, there has been further work on formative assessment, and what is required to make this work in the classroom. James Popham's book (2008) describes 'learning progressions' which require the teacher to have a thorough understanding of how learning takes place and which skills and concepts are essential pre-requisites for particular learning. This highlights a difficulty with implementing effective formative assessment, in mathematics as well as other subjects, it requires a thorough understanding of the subject content, the pedagogies required for conveying that content, and of the ways in which students learn. Bennett (2011) expands on this issue by highlighting that effective formative assessment practice is domain specific, that is, it is not the same in different subject areas. He goes on to state that a key implication of this is that 'a teacher who has weak cognitive-domain understanding is less likely to know what questions to ask of students, what to look for in their performance, what inferences to make from that performance about student knowledge, and what actions to take to adjust instruction' (p. 15).

Bennett (2011) goes on to highlight another issue important to consider here. That is the interaction between formative and summative assessment, what he terms 'the system issue'. He points out (quoting Pellegrino et al., 2001) that the different components of an education system must be coherent if they are to work together effectively. This coherence applies to the use of summative and formative assessment. Bennett suggests that the constrained nature of some summative assessments limits classroom practice, and that in turn this limits the potential of formative assessment to lead to significant improvements.

European countries provide national guidelines on the use of various forms of classroom assessment in mathematics. Figure 3.1 details the forms of assessment that are advocated for formative purposes.

Figure 3.1: National level guidelines on assessment methods to be used for formative purposes in mathematics, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

UK (¹) = UK-ENG/WLS/NIR

Country responses on the existence of national level guidelines for project, portfolio, ICT or self/peer-based assessment for formative purposes provide a mixed picture. Estonia and Liechtenstein note that guidelines are given, but not specifically for mathematics. In half of the countries, there are no guidelines on any of the mentioned types of assessment. Among these countries, the Czech Republic and Finland observe that central education authorities focus on assessment outcomes, rather than on methods, and the Flemish Community of Belgium and Sweden note that the choice of assessment method is the prerogative of individual teachers or schools.

In the **United Kingdom (England, Wales and Northern Ireland)** also, there are no national guidelines for formative assessment specifically of mathematics. However, in Wales and Northern Ireland, general guidelines on 'assessment for learning' across the curriculum are provided. In England, non-statutory guidance exists for formative assessment in mathematics, but the government does not prescribe or enforce any particular approach to formative assessment.

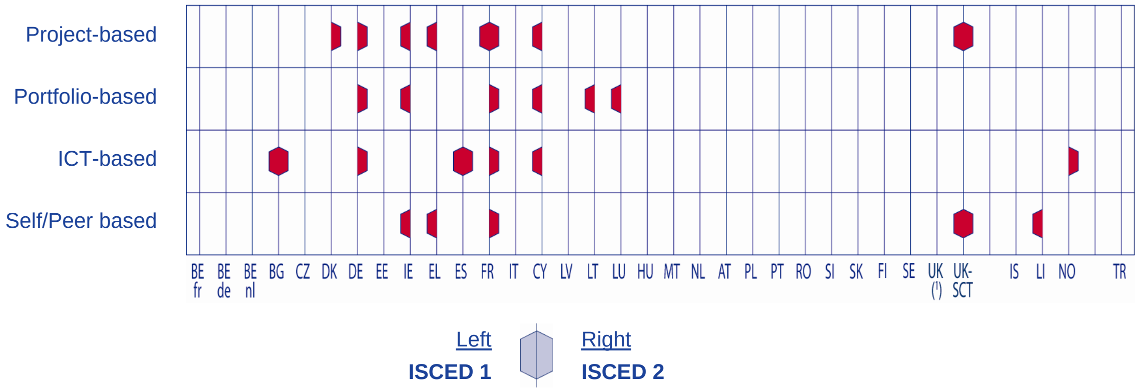
In the **United Kingdom (Scotland)**, building on the expertise gained through 'assessment for learning', an assessment document to support the new curriculum has been published ([1]). To support and exemplify key aspects, an on-line national assessment resource that highlights good assessment practice in all areas of the curriculum, including concrete examples for mathematics, is currently under construction. It will showcase how schools have supported effective learning and teaching through well planned assessment procedures and will also allow teachers to share nationally how they married effective learning and teaching with rich assessment procedures.

As shown in Figure 3.2, guidelines from authorities for the summative use of project, portfolio, ICT or self/peer-based assessment are even less common than for formative use. France is the exception, where the resource documents ([2]) are very explicit and give numerous examples of all types of assessment – diagnostic, formative, summative and also self-evaluation.

([1]) http://www.ltscotland.org.uk/buildingyourcurriculum/policycontext/btc/btc5.asp

([2]) For ISCED1, see http://www.education.gouv.fr/cid48791/troisieme-note-synthese-sur-mise-oeuvre-reforme-enseignement-primaire.html, for ISCED 2, see http://igmaths.net/

Figure 3.2: National level guidelines on assessment methods used for summative purposes in mathematics, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

UK (1) = UK-ENG/WLS/NIR

PISA 2003 also investigated the use of different forms of assessment. According to the responses of school principals, the most common methods of assessment were teacher-developed tests and student assignments/projects/homework (OECD 2004, pp. 418-420). In most European countries, the percentages of 15-year-old students whose principals reported using each of these methods of assessment more than three times a year was about 80 % or more. A handful of European countries were markedly different, however. In Turkey, only 40 % of students had principals who reported using teacher-developed tests more than three times a year. The comparable figure for Denmark was 65 % and for Ireland, 74 %. Similarly, only 15 % of students in Greece and 36 % of students in Turkey had principals who reported using student assignments as a method of assessment at least three times a year. According to PISA data, student portfolios were also used more commonly than standardised assessments. This form of assessment was especially common in Denmark, Spain and Iceland. In these countries, more than 80 % of students were enrolled in schools where student portfolios were used at least three times a year.

The use of calculators in mathematics assessment is recommended or prescribed in around half of European countries (see also Chapter 2.3 on use of calculators for teaching). Some countries such as Malta and Liechtenstein recommend calculator use at secondary level only, and the United Kingdom (Scotland) highlights the need for restricted use of calculators in the assessment process to promote the development of basic skills. Portugal appears to be the only country where the type of calculator used is stipulated.

3.2. The role of national testing

What is taught in schools is often determined by what is assessed; particularly where the assessment results are used for high stakes purposes. The nature of the assessments is said to determine the nature of teaching and learning and may limit the use of more effective or innovative modes of teaching (Burkhardt, 1987; NCETM, 2008). Looney (2009, p. 5) states that the high stakes associated with results from some national tests can 'undermine innovative approaches to teaching, including formative assessment'.

The EACEA/Eurydice (2009) report found that national testing of students is a widespread practice in European education systems. The results of national tests are used to award certificates, and/or to monitor and evaluate schools or the system as a whole. National tests are less frequently used to for formative purposes i.e., to identify the specific learning needs of students. Depending on the objectives, tests can be compulsory for all students or optional, or they could be administered on a sample of students.

The report showed that some countries assess only a few subjects – viewed as the core curriculum, while others test a broader range. Mathematics is tested even where only two or three subjects are routinely assessed. The focus of assessment may vary, for example it may be based on a broad definition of mathematics or it may focus on the core skills of numeracy, or it may have a more applied approach in terms of mathematical competence.

During the 2010/11 school year only Belgium (German-speaking Community), the Czech Republic, Greece and the United Kingdom (Wales) have not held any national tests for students of compulsory school age (although the Czech Republic plans to introduce tests from 2013). While some European countries, such as Malta and Norway, hold national tests in mathematics in almost every school year, the majority of countries administer national tests only two or three times during the period of compulsory schooling (EACEA/Eurydice, 2009). In rare cases, as in Belgium (Flemish Community), these tests do not address the achievement of individual students but are used only for system-monitoring.

The rise of national testing is confirmed by the recent launch of new tests in a number of countries:

> Starting from the 2010/11school year, **Liechtenstein** has introduced national tests in mathematics which are compulsory for all students in years 3 and 5 of primary and year 7 of secondary level. In **France**, since 2009, all pupils in years 2 and 5 of primary (CE1 and CM2) have been taking new national tests in mathematics. Other countries have also recently added new national tests in mathematics in specific school years, such as the national test in **Italy** in year 10, the 'Nationally Coordinated Examinations' in year 10 in **Iceland**, and the voluntary tests in numeracy and arithmetic skills at years 1 and 3 in **Norway**.

Despite an apparent increase in national testing in some European countries, international survey data reveals limited relevance attached to this assessment instrument by teachers. TIMSS 2007 results showed that, typically, teachers of eighth grade students gave only moderate emphasis to national or regional achievement tests, with little or no emphasis on this source of information for 30 % of students and some emphasis for 40 %. Even fewer students had teachers who placed major emphasis on national or regional tests in monitoring student progress in the Czech Republic, Italy, Cyprus, Lithuania, Hungary, the United Kingdom (Scotland) and Norway (Mullis et al. 2008, p. 309). In most of these countries, there either are no national tests or the tests are based on a sample of students and therefore teachers have no opportunity to exploit the results of this assessment method.

3.3. Mathematics in upper secondary education

The importance assigned to the acquisition of a certain level of skills and competences in mathematics by the completion of upper secondary education is illustrated by data in Figure 3.3 on the proportion of students taking school leaving examinations in this subject.

Mathematics is a compulsory subject for all students in the examinations at the end of upper secondary school in around half of the countries. In other countries (Austria, Italy, the Netherlands, Luxembourg and Romania), only the students from certain branches of education are obliged to take examinations in mathematics, although the proportion of students in this category can be high, for example in the Netherlands it is 85 % and in Luxembourg 90 %. In countries where mathematics is only an elective subject (Bulgaria, Estonia, Lithuania, Malta, Slovakia, Finland, the United Kingdom (Scotland) and Norway, it can still be taken by a significant number of students, as for instance in

Lithuania, Slovakia and the United Kingdom (Scotland) where around half of all students choose to sit school leaving examinations in mathematics.

Figure 3.3: Inclusion of mathematics in school leaving examinations at the end of upper secondary education by country, 2010/11

	Compulsory maths exam for:		Exam in maths as an elective subject
	all students	students in a specific branch	
BE fr	●		
BE de	●		
BE nl	●		
BG			● (10 %)
CZ		●	●
DK	●		
DE	●		
EE			●
IE	●		
EL	●		
ES	NA	NA	NA
FR	●		
IT		● (25%)	
CY	●		
LV	●		
LT			● (50 %)
LU		● (90 %)	

	Compulsory maths exam for:		Exam in maths as an elective subject
	all students	students in a specific branch	
HU	●		
MT			●
NL		● (85 %)	
AT	●(for AHS)		● (for BHS)
PL	●		
PT	●	●	
RO		●	
SI		● (general upper education)	● 40 % (vocational education)
SK			● (58 %)
FI			●
SE	●		
UK-ENG/ WLS/NIR	● (until age 16)		●(students aged 16-18)
UK-SCT			● (>50 %)
IS		●	
LI	●		
NO			●
TR	NA	NA	NA

Source: Eurydice.

Country specific notes

Spain and **Turkey**: No school leaving examinations in mathematics exist, but there are university entrance examinations.
Austria: AHS (academic secondary schools); BHS (upper level secondary vocational and technical schools)

The United Kingdom and Hungary point to the fact that there is a high academic value placed on mathematics in terms of accessibility to further study and future careers. Further emphasis is placed by schools on the mathematics examinations taken by students in England, Wales and Northern Ireland at age 16. Although this is not the end of upper secondary education, the results of these examinations are part of the criteria used to benchmark the performance of schools. Despite the high value placed on mathematics attainment, it is interesting to note that the four regions of the United Kingdom were found to have some of the lowest levels of participation in mathematics beyond age 16 (Hodgen et al., 2010).

3.4. Use of mathematics assessment data

A number of countries report that various reforms to mathematics education are driven or supported by the analysis of the results of international surveys and nationally standardised tests. This section concentrates on the use of national test results for the improvement of mathematics education at national and school level.

In broad terms, test results serve to prompt debate about the effectiveness and appropriateness of the mathematics education system. Schools are often encouraged to analyse their students' results and to compare them to the national average. National information reveals that curriculum development and teacher training and professional development are the areas that most often undergo changes due to the influence of national test results. Moreover, national test results are used in around half of the countries for policy formulation at national level.

Belgium (Flemish Community), Denmark, Estonia, France, Ireland, Lithuania, Latvia and **Romania** review existing curriculum documents in the light of national tests and examinations. Education authorities in **Bulgaria** use the results to target resources on low achievers by developing a further education programme for this group. **Belgium (French Community), Estonia, Lithuania** and **Liechtenstein** use the results to improve the areas of teaching that need further support or development, for example, through teacher-training or CPD programmes, or the launch of projects on innovative methods. In **Spain**, results from the general diagnostic evaluations are included in the National System of Education Indicators that is used as one of the basis for designing improvement measures.

In certain cases, national test results are not directly used as a source for national level improvements or policy formulation.

In **Malta, Poland** and **Iceland**, it is up to individual teachers and/or schools to interpret the results and to decide how to react to the feedback from national tests. In the **Netherlands** results can be a reason for relevant bodies including subject-associations (NVORWO, the Commission for Standards for Mathematics Education and NVvW, the Association of Teachers of Mathematics) and research institutes to consider modifying teaching approaches.

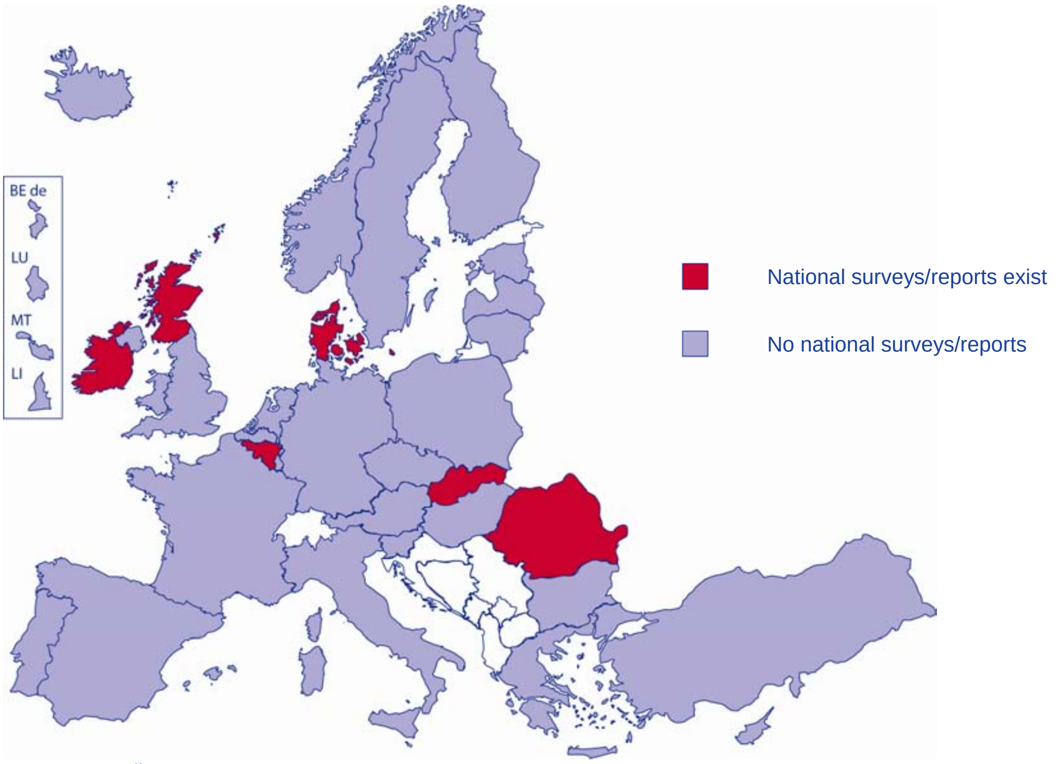
Finally, international surveys provide some insight about the regular use of mathematics assessment data. PISA 2003 asked school principals about the common uses of mathematics assessment data. The results showed that at the school level the assessment data were mostly used to inform parents about their child's progress. The assessment data were also commonly used to make decisions about student retention or promotion to the next class and to identify aspects of teaching or the curriculum that could be improved. Less common uses were to support decisions about grouping students, for benchmarking against national standards, for monitoring teacher effectiveness and for making comparisons with other schools (OECD 2004, pp. 421-424).

3.5. National surveys and reports for evidence-based policies on assessment

Current national policies and debates regarding assessment often concentrate on the move from an overreliance on summative assessment to a more balanced approach (Malta and the United Kingdom (Scotland)). The Czech Republic, Estonia and Spain emphasise the need for a change in teachers' assessment culture and for appropriate training on using various assessment instruments for formative purposes. Other countries like the Netherlands, Austria and Slovenia focus their efforts on redesigning the examination system at the end of upper secondary education.

A minority of countries have been focusing on how teachers select their methods for assessing students in mathematics. Clearly, this information is useful to both to inform new policy developments and evaluate the success of previous initiatives.

Figure 3.4: National surveys/reports on teachers' selection of methods to assess students in mathematics, 2010/11



Source: Eurydice.

As shown in Figure 3.4, only a minority of European education systems survey or report on teachers' selection of methods to assess students in mathematics. The reports that have been published identify a number of challenges and areas for improvement.

In **Denmark**, the Danish Evaluation Institute reports on assessment (as well as teaching methods and content). The most common form of assessment (used by 42 % of teachers) for formative purposes are teacher-parent consultations with students present. This was followed by the use of tests by 24 % of teachers, and then by conversations between teacher and student, used by 18 % of teachers. The 2006 report also highlights the need to strengthen awareness of the potential of assessment and the need to develop different tools to support it (3).

In **Ireland**, there are a number of reports that include information on the use of assessment in schools. For example the 2009 report into the national assessment of mathematics and English reading (4) found that:

- Most children in the fourth and eighth year of primary school are tested using standardised mathematics tests. For the 2008/09 school year, 5 % of teachers of pupils in the fourth year of schooling and 10 % of teachers of pupils in the eighth year of schooling did not anticipate that standardised tests of mathematics would be administered.
- Teacher questioning was the most frequently used form of non-standardised testing.
- Roughly 90 % of pupils attended schools whose principal teacher agreed that aggregated standard tests results in maths were discussed at staff meetings and used to monitor school-level performance. Less than

(3) 'Matematik på grundskolens mellemtrin – skolernes arbejde med at udvikle elevernes matematikkompetencer', Danmarks Evalueringsinstitut (The Danish Evaluation Institute), 2006. http://www.eva.dk/eva/projekter/2005/arbejdet-med-at-udvikle-elevernes-matematikkompetencer/projektprodukter/matematik-paa-grundskolens-mellemtrin-skolernes-arbejde-med-at-udvikle-elevernes-matematikkompetencer

(4) http://www.erc.ie/documents/na2009_report.pdf

three-quarters of pupils were in schools whose aggregated results were used to establish teaching and learning targets. The most common use of test results at the individual level was to identify pupils with learning difficulties.

Lithuania uses information gathered from national testing and reports by the National Agency for Assessment of Schools and notes that teachers often do not fully understand the concept of formative assessment and that they give students low quality feedback. Moreover, teachers and students' views on the quality of assessment often differ significantly and the larger these differences, the lower the students' achievements ([5]).

Summary

The evidence in this chapter shows the importance of classroom assessment across European countries and the prominent role teachers play in preparing and administering it. It therefore also indicates a potential need for guidelines and support measures for teachers with regard to assessment issues.

Both formative and summative assessment are viewed as important in countries across Europe with the amount of national testing increasing as well as the development of a number of policies to support formative assessment. Mathematics is seen as a key focus of testing with its inclusion in a large proportion of countries' national testing systems, even in those where only a small number of core subjects are tested. A number of countries explicitly mention the high status associated with success in mathematics at the higher levels.

However, there appears to be little prescription provided on the nature of classroom assessment in different countries, with teachers being free to choose how they collect evidence of progress. Some countries (the United Kingdom – England and Scotland) provide support at a central level for classroom assessment, although the materials and resources are for optional use. Both TIMSS and PISA results reveal that the use of teacher tests is a widespread practice in both primary and secondary schools.

As might be expected, there is much greater prescription regarding the assessment of mathematics through national testing, with mathematics tests being compulsory in a large majority of cases. The results from assessments are used to improve education generally, and for a wide range of more specific purposes including to target resources at particular groups of students; to inform curriculum reviews; and to inform approaches to teacher professional development, although not all countries use the assessment results in a structured way.

Only a minority of countries claim to monitor the use of assessment methods. This might be understandable with regards to national testing, as this is often compulsory and results will be available at a national level, although less understandable for classroom assessment. As the research data shows, the effective use of classroom assessment can have a large impact on attainment, but it is not easy for teachers to do it well. This is, therefore, an area where greater monitoring might be advantageous.

([5]) NMVA (National Agency for Evaluation of Schools), 2010. Review of Quality Evaluation Activities of General Education Schools During 2007-2008 Year Period. Informacinis leidinys "Švietimo naujienos" 2010, No.1 (290), priedas, p.p. 1-16. (In lithuanian); Ministry of Education and Science, 2008. National Student Achievement Study 2006: Grades 6 and 10: Analytic Report. Vilnius: ŠMM. Available at: http://www.upc.smm.lt/ekspertavimas/tyrimai/2006/failai/Dalykine_ataskaita_2006.pdf [Accessed: 11 June 2011].

CHAPTER 4: ADDRESSING LOW ACHIEVEMENT IN MATHEMATICS

Introduction

Low achievement in mathematics is a common concern for all European countries. It is an issue associated not only with the effectiveness of teaching and learning, but also with providing an equitable system of education. A range of approaches have been developed to support under-performing students and to attempt to close the persistent gap between the highest- and lowest-achieving students. By bringing together research, survey results and information on national policies this chapter outlines national approaches and current practice for addressing low achievement in, and outside the mainstream classroom. In this analysis low achievement refers to student performance that is below the expected level of attainment. Under-performance occurs for a wide variety of reasons. However, this analysis focuses on school-related factors and does not address those linked to learning disabilities such as dyscalculia ([1]), and does not address the provision of support exclusively related to special needs education.

Section 1 concentrates on the tools used at national level to formulate evidence-based policies on low achievement. Section 2 presents an overview of research results on effective measures to tackle under-performance in mathematics while Section 3 outlines the main elements of national policies to raise achievement. Finally, Section 4 examines the use of specific forms of support for low achievers across Europe.

4.1. Evidence-based policy on low achievement

The results of international surveys, as well as other research evidence point to the fact that low achievement in mathematics is a complex phenomenon (Mullis et al., 2008; OECD, 2009b; Wilkins et al., 2002; Chudgar and Luschei, 2009). At national level, collecting evidence on performance trends, factors contributing to underachievement, and effective approaches for raising attainment can provide significant support to the policy making process. However, as Figure 4.1 shows, half of all countries in Europe do not conduct any such surveys or reports. Even less common are independent evaluations of support programmes for low achievers.

Countries often use analyses of PISA and TIMSS data to assess mathematics performance and identify the reasons for students' low achievement. In some cases, these analyses are supplemented by reports based on results from national standardised tests. In both cases, conclusions point to the fact that underachievement in mathematics occurs for a number of reasons linked to home background and school-related factors that often reinforce each other (see 'Achievement in mathematics: evidence from international surveys').

> In the **Flemish Community of Belgium**, for instance, the *Periodieke Peilingen* (Periodical national assessment of performance), from 2008/09, shows that underachievement in mathematics is linked to the language spoken at home where it is different to the language of instruction; low intrinsic motivation; and low social/economic background ([2]).
>
> In **Ireland**, the analysis of the results of the 2009 National Assessments of Mathematics and English Reading ([3]) concluded that lower scores were linked to large family size, parental unemployment, membership of the traveller community, coming from a lone-parent family, and speaking a language at home other than the language of instruction. Positive factors linked with test scores included the wide availability of books and educational resources at home;

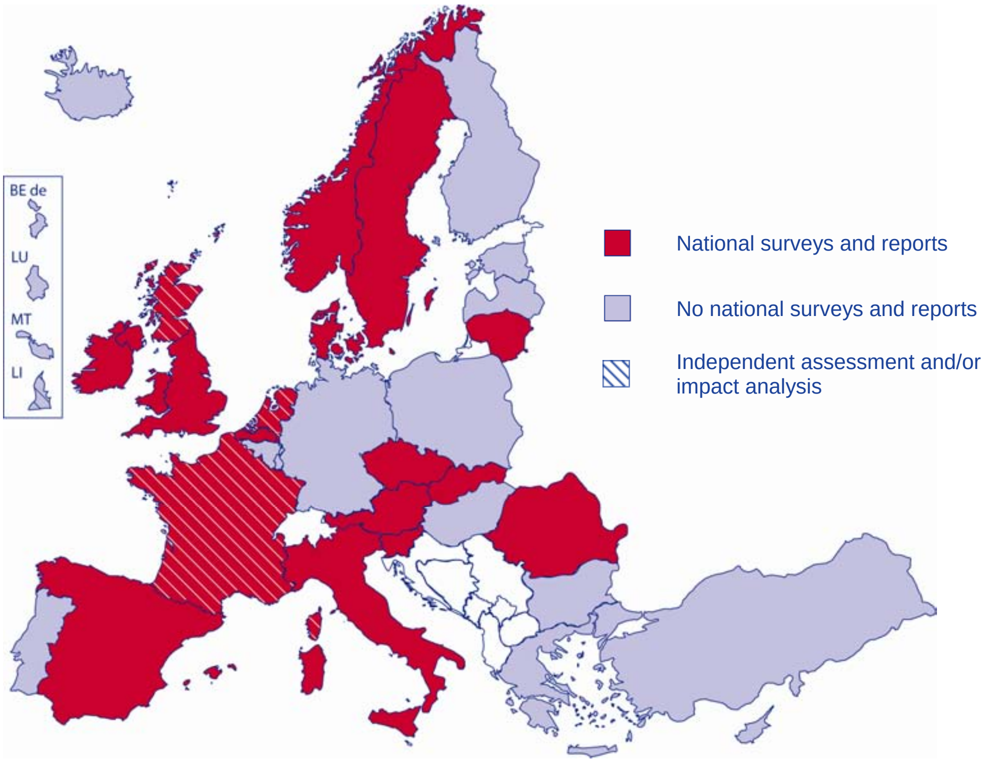
([1]) A condition that affects the ability to acquire arithmetical skills

([2]) http://www.ond.vlaanderen.be/dvo/peilingen/basis/Brochure_peiling_wisk_bis.pdf

([3]) http://www.erc.ie/documents/na2009_report.pdf parental confidence in assisting with homework; and higher mathematics self-concept (how mathematics learners perceive themselves). Teacher characteristics linked with higher test scores included teaching experience, additional qualifications, and infrequent use of table books for maths.

Similarly, in **Spain**, a report on the results of the first General Diagnostic Evaluation carried out in 2009 with students in the fourth year of primary education shows that there is a strong correlation between achievement level in mathematics and four out-of-school factors: parents' level of education and occupation; the number of books at home; and the availability of other resources at home such as a quiet place to study and an internet connection.

Figure 4.1: National surveys and reports on low achievement in mathematics, 2010/11



Source: Eurydice.

Some national analyses of the causes of underachievement in mathematics highlight additional factors that have significant importance in specific national contexts.

In **Italy**, the report of SNV, *Servizio Nazionale di Valutazione* (National Assessment Programme), for the year 2010 underlines regional differences between the northern and southern parts of the country which appear to increase in lower secondary education. Moreover, while performance is fairly uniform in the north, it varies greatly in the south. On the other hand, non-Italian students obtain substantially lower results, in a much more geographically uniform way than Italian students.

National reports in **Romania** have identified several factors that negatively affect performance in rural schools. These are mainly related to the high turnover, low motivation (social and financial) and the inadequate mathematics qualifications of teachers in these schools, as well as the grouping of pupils in mixed age classes at primary level ([4]). Since 2010 these structural and personnel problems have been addressed to a varying degree. In particular, the

([4]) http://proiecte.pmu.ro/web/guest/pir
http://didactika.files.wordpress.com/2008/05/modul-adaptare-curriculum-la-contextul-rural.pdf
http://didactika.files.wordpress.com/2008/05/modul-recuperarea-ramanerii-in-urma-la-matematica.pdf

practice of forming mixed age classes has been discontinued and 600 rural teachers have obtained an additional university qualification in teaching mathematics.

In **Sweden**, a recent report of the National Agency for Education, based on a systematic review of international and Swedish research, points out that performance is also influenced by structural factors such as increased decentralisation of school management, resource allocation and streaming, as well as by in-class factors such as peer-group effects and teacher expectations (Swedish National Agency for Education, 2009).

In addition, national studies provide data on problematic subject content and mathematical skills. In Ireland, Lithuania, Romania and Slovenia, for instance, algebra, mathematical communication, and problem solving in context have been identified as common problematic areas for students. Not surprisingly, the same content areas presented difficulties for teachers. The 2006 EVA *Evaluation on Mathematics* reported that Danish teachers found communication, problem-solving, and understanding the role of mathematics in context particularly difficult objectives to accomplish ([5]).

In identifying 'what works' for low achievers in the past ten years, independent assessment or impact analysis of support programmes have been undertaken in France, the Netherlands, the United Kingdom and Lichtenstein.

In **France**, the Court of Auditors published an extensive report *National education and the objective of success for all pupils* in 2010 (Cour des comptes, 2010) which is based on field studies and interviews with practitioners and experts. The report concludes that the national education system needs to improve its efficiency and effectiveness in order to provide more equitable education. It also emphasised that existing tools for tackling low achievement in mathematics have not been providing satisfactory results. An inspection report from 2006 had already put forward recommendations for improving the implementation of the *Programmes personnalisés de réussite scolaire* at primary and secondary level. They include the need to harmonise divergent and sometimes contradictory practices; improve the criteria for selecting participating students; set precise and realistic objectives for improvement; and provide targeted training for teaching staff and others (Chevalier-Coyot et al., 2006).

In the **United Kingdom (Scotland)**, the impact of the 'Early Years and Early Intervention' initiative which suggests effective support measures to raise achievement generally is currently being monitored. Early intervention to help young children develop confidence with number, especially through parental involvement is a key feature of the document ([6]).

4.2. Key research findings on effective measures to address low achievement

The importance of out-of-school factors including students' socio-economic background and the educational level of parents or the language spoken at home cannot be overstated. Significantly reducing the proportion of low achievers in mathematics, therefore, would require a combined approach that simultaneously targets a range of factors both in and out of school. The following sections, however, concentrate primarily on factors that can be directly influenced by education policies.

In order to be successful, strategies to address low achievement need to be embedded within all aspects of learning and teaching, including curriculum content and organisation, classroom practices and teacher education and training. Moreover, a comprehensive approach would comprise measures that are suitable for all students, but benefit underperforming students in particular; it should also

([5]) 'Matematik på grundskolens mellemtrin – skolernes arbejde med at udvikle elevernes matematikkompetencer', Danmarks Evalueringsinstitut (The Danish Evaluation Institute), 2006, available at http://www.eva.dk/projekter/2005/arbejdet-med-at-udvikle-elevernes-matematikkompetencer/projektprodukter/matematik-paa-grundskolens-mellemtrin-skolernes-arbejde-med-at-udvikle-elevernes-matematikkompetencer

([6]) http://www.scotland.gov.uk/Publications/2008/03/14121428/6 include arrangements for providing targeted support for students with individual needs both inside and/or outside the normal classroom.

Responding to the diverse needs of learners

While acknowledging the common learning needs of all students in the classroom, teachers should give attention to students' individual needs and learning styles and adjust their teaching accordingly (Tomlinson, 2003; Tomlinson and Strickland, 2005). Research evidence indicates that accommodating the diverse range of students' learning needs in terms of readiness to learn, interest, and individual learning profiles has a positive impact on achievement and engagement with mathematics (Tieso, 2001, 2005; Lawrence-Brown, 2004).

Emphasising the relevance of mathematics

Teaching methods should address the perceptions that mathematics is difficult, abstract or uninteresting and not relevant to real life. One way of doing so is to organise lessons around 'big ideas' and interdisciplinary themes that help establish connections with everyday life and other subjects. This approach is at the core of the well-established 'Realistic Mathematics Education' programme in the Netherlands (Van den Heuvel-Panhuizen, 2001).

Early interventions at primary level

The first two years of schooling provide the foundations for further mathematics learning. Identifying difficulties at this stage can prevent children from developing inappropriate strategies and misconceptions that can become long-term obstacles to learning (Williams, 2008). Children at risk should be specifically targeted, including with prevention programmes at pre-school level. Early intervention can also combat the development of anxiety which can become a significant factor among older students (Dowker, 2004).

Focusing on individual weaknesses

An extensive review of research evidence on *'What works for children with mathematical difficulties'* has concluded that 'interventions should ideally be targeted towards an individual child's particular difficulties' (Dowker, 2004).

Individual support has proven to have significant impact on children's performance (Wright et al., 2000, 2002). However, because of the heterogeneity of approaches, it is difficult to compare intervention schemes and their effectiveness. Nevertheless, it could be assumed, that 'in most cases, if interventions start early and concentrate on specific weaknesses, they might not need to be very long or intensive' (Dowker, 2009).

Motivational factors

An additional constraint on progress in mathematics that is especially valid at secondary level is the issue of motivation (Chapter 5). Teachers need to set and communicate high expectations and encourage the active participation of all students (Hambrick, 2005). Together with parents, teachers should emphasise the value of effort against a certain resignation that success in mathematics is largely due to inherent ability (National Mathematics Advisory Panel, 2008). They also need to develop 'soft skills' such as connecting with students, engaging them, and managing the classroom in a way that can prevent disengagement at secondary level (Gibbs and Poskitt, 2010).

Increasing parental involvement

Parents should be encouraged to help their children to learn and enjoy mathematics. Moreover, the involvement of parents is vital for the success of intervention programmes (Williams, 2008). At the same time, in view of the data on the level of adult numeracy skills, it should be recognised that some parents might not be able to provide adequate support for their children's learning.

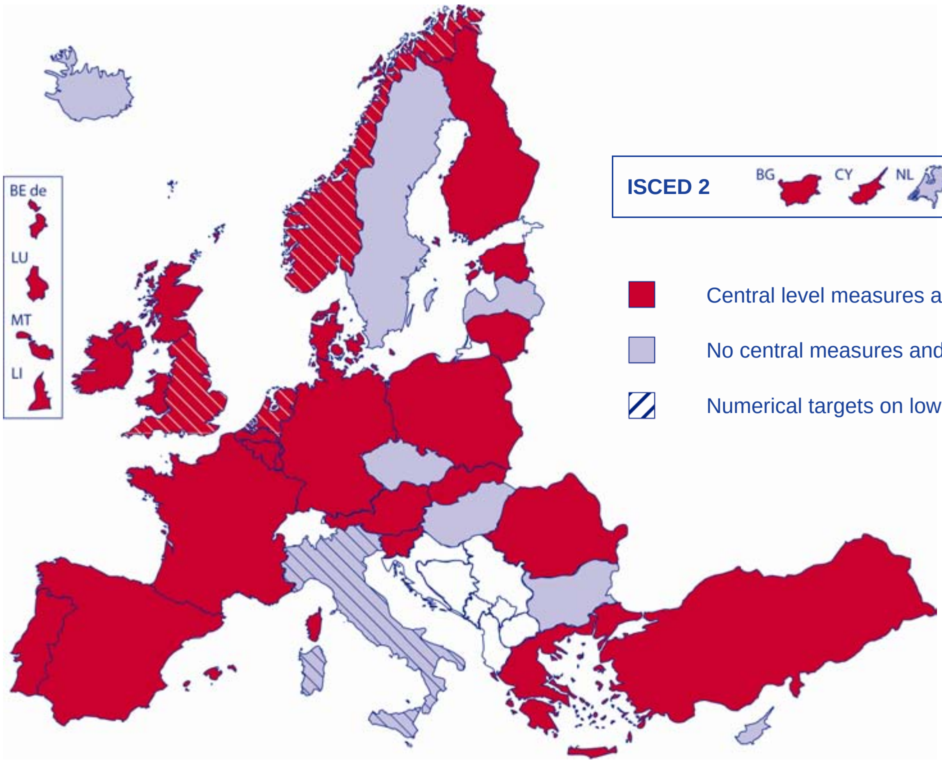
Links to literacy problems

Achievement in mathematics closely correlates with performance in other key areas such as reading literacy and science (OECD 2010d, p. 154). Research has demonstrated the relationship between mathematics learning and language factors such as reading comprehension (Grimm, 2008). The interrelation between literacy and numeracy problems, in particular, should be considered when planning support (Williams 2008, p. 49).

4.3. National policies to raise achievement

In the majority of European countries, central education authorities prescribe or recommend support measures or assist schools and teachers in implementing measures to address students' difficulties in mathematics (see Figure 4.2).

Figure 4.2: National level guidelines in addressing low achievement in mathematics, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

National level involvement in tackling low achievement varies in both the level of compulsion imposed on schools and the degree of detail in guidance documents. Measures often apply to the teaching of both mathematics and the language of instruction, and sometimes to other subjects as well. They also usually differentiate between primary and secondary education.

Central level measures range from comprehensive national programmes which are compulsory (Estonia and Spain) to targeted support for a limited number of activities such as in-service professional development on low achievement (Belgium – German-speaking Community), or the provision of data banks of educational resources in mathematics (Finland). The following national examples can serve as an illustration of existing national involvement in this area.

Several countries report that strategies to tackle low achievement have been developed at national level. These strategies translate general policy objectives into specific measures and activities that are to be applied throughout the education system.

In **Estonia**, one of the objectives of the General Education System Development Plan for 2007-2013 is to create opportunities for individualised learning that take into account students' different learning abilities with a view to reducing grade retention and school drop-out figures '. Mathematics test results are being analysed by an independent research group and are published annually. Specific approaches that are prescribed include the use of an individualised curriculum, supplementary classes, consultations, remedial groups *(parandusõpe)* and counselling of parents.

In **Ireland**, in accordance with the Learning Support Guidelines issued by the Department of Education, early detection and intervention and differentiated teaching are the key approaches promoted in classrooms. The use of these strategies complements the learning support provision (i.e. supplementary teaching) delivered by learning support teachers provided mainly on the basis of withdrawing students from their normal lessons, although there is a growing emphasis on the provision of support to individual students within classrooms. In-class cooperative support, one-to-one withdrawal and team teaching also feature.

In **Spain**, the Ministry of Education's Action Plan 2010-2011, developed in collaboration with the Autonomous Communities, is organized around 12 major goals that emphasize the 'achievement of educational success for all students, as well as the equity and excellence of the education system' through the adoption of 'basic competences'. In primary education, regulations establish that support mechanisms should be implemented as soon as learning difficulties are detected. They are both organisational and curricular and consist of individual tuition in the ordinary group, flexible grouping or curricular adaptations. In lower secondary education, regulations emphasize attention to diversity and responsiveness to the specific educational needs of students. The measures that are prescribed include the offer of optional subjects, reinforcement measures, adapting the curriculum, flexible grouping and split classes.

In **Poland**, the Ministry of National Education launched a wide-ranging programme of student support in 2010 that includes a focus on low achievement and high risk groups. Recommended forms of support include remedial and compensation classes, diagnosis of difficulties in pre-school and primary school, and individualised career guidance.

In **Norway**, the main elements of the national policy to reduce low achievement are based on early intervention, national tests and mapping (diagnostic) tests, and the integration of basic mathematics skills in all subject curricula. The national strategy, *Science for the future: Strategy for strengthening mathematics, science and technology (MST) 2010 – 2014* ([7]), and the National Centre for Mathematics Education (see Annex) are important agents in promoting mathematics education.

In other countries, central authorities issue relatively general recommendations that leave the choice of practical measures to the discretion of teachers.

In the **United Kingdom** (**Scotland**), the government has recently published a document asking teachers to reflect on how they can best support young people who are challenged by certain aspects of education. Teachers of mathematics will be expected to ensure that learning and teaching approaches align with key aspects of the document ([8]). Although the central government does not recommend specific approaches, a number of education staff are trained in the 'Maths

([7]) http://www.regjeringen.no/en/dep/kd/documents/reports-and-actionplans/Actionplans/2010/science-for-the-future.html?id=593791

([8]) http://www.hmie.gov.uk/documents/publication/cuisa09.html

Recovery' approach to supporting students with mathematics difficulties. There is a well established support group of teachers in Scotland which promotes Maths Recovery methods ([9]).

In **Denmark**, the Ministry of Education has produced a special document that contains several recommendations on how to address learning difficulties in mathematics. It recommends that mathematics teachers carefully observe low achievers, engage in a dialogue with them and focus on what they can do, rather than on what they cannot do. Beyond assigning such students easier tasks, teachers should also guide them towards new strategies to cope with their difficulties.

In some countries where schools have a significant degree of autonomy, the central education authorities nevertheless provide some support to teachers and schools for addressing low achievement in mathematics.

In **Finland**, the core curriculum contains guidelines on general support for students. The most common approach is early detection and support. The Ministry of Education organises targeted in-service teacher training and maintains a web-site ([10]) with information on the most common learning problems in mathematics in the early school years. The site provides access to computer-assisted instruction methods for mathematics (Number Race, Ekapeli-Matikka and Neure). In addition, specific tests for the diagnosis of learning problems are available for purchase from private companies.

In **Belgium** (**Flemish Community**), the government is providing support to low achievers via the national program of 'gelijke kansen' (equal opportunities). The implementation of support is determined at school level but the inspectorate is monitoring the outcomes of any measures taken.

In **the Netherlands**, the Ministry limits its involvement to support for research projects and meetings of groups of experts. The main focus of these activities is on promoting individualised and remedial teaching and increasing the involvement of parents.

Only the central authorities in the Czech Republic, Italy ([11]), Latvia, Hungary, Sweden and Iceland do not provide any guidelines or support to teachers and schools in tackling low achievement in mathematics in either primary or lower secondary education. In these countries, depending on the model of decentralisation in place, each school and/or municipality is responsible for the design and implementation of such measures. In Sweden, for instance, school providers are responsible for providing all the tools and support mechanisms necessary for the completion of the achievement goals that have been set for each education level.

National targets on mathematics achievement

Using results from international surveys, and in particular PISA, to measure progress in mathematics achievement, is an approach that has been taken up at European level (European Council, 2008). However, it appears that this policy is not widespread at national level, despite the widely reported use of results from international surveys. Although several countries have set national objectives regarding low achievement in mathematics, the majority of these are not numerical targets and are not linked to performance in international or national tests. Usually these targets refer to standards or competence levels that are to be achieved at a certain stage or to objectives linked to reducing early school leaving.

([9]) http://www.mathsrecovery.org.uk

([10]) www.lukimat.fi

([11]) In Italy, only upper secondary schools are required by law to activate support measures for low achievers.

In **France**, for instance, by age 16, students must acquire specific competences in mathematics in line with the common competence framework. In **Sweden**, specific competence levels are to be achieved in the third, sixth and ninth year of schooling. In **Germany** and **Estonia** targets on mathematics achievement are linked to strategies to combat early school leaving.

Only Italy, the Netherlands, the United Kingdom (England) and Norway have set national targets on low achievement that are based on results in international and/or national standardised tests.

In **Italy**, although the Ministry of Education does not provide guidelines on how to tackle low achievement, it has set explicit targets for lowering the number of low achievers in mathematics. The national target is to reduce the proportion of low-achieving Italian students in the PISA test (i.e. the percentage of students with proficiency in mathematics at level 1 and below) to 21 % in 2013. For comparison, in PISA 2009, this figure was 25 % (see 'Achievement in mathematics: evidence from international surveys').

In **Ireland**, comprehensive national targets for reducing low achievement in mathematics will be implemented in the period 2011-2020. These targets have been outlined in *Better literacy and numeracy for children and young people: A draft national plan to improve literacy and numeracy in schools* (November, 2010) and include the following:

- Reduce the percentage of students performing at, or below level 1 (minimum level) in the National Assessment of Mathematics by at least 5 % in the fourth and eighth years of primary school.
- Increase the percentage of students performing at, or below levels 3 and 4 in the National Assessment of Mathematics by at least 5 % in the fourth and eighth years of primary school.
- Increase the percentage of students achieving the equivalent of Grade C or above in the Mathematics ordinary level examination in the Junior Certificate examination or its equivalent from 77 % to 85 %.
- Increase the percentage of students taking the Higher Level mathematics examination in the Junior Certificate examination or its equivalent to 60 %.
- Increase the percentage of students taking the Higher Level mathematics examination in the Leaving Certificate to 30 %.

4.4. Types of support for low achievers

A variety of approaches to support students with difficulties in mathematics are employed both inside and outside the regular classroom (Dowker et al., 2000; Gross, 2007).

Methods that are used within the classroom include ability grouping (see Chapter 2), individualised teaching, or, less often, the use of teaching assistants. Outside the classroom various types of support are provided including peer-assisted learning, group collaboration, and individual support.

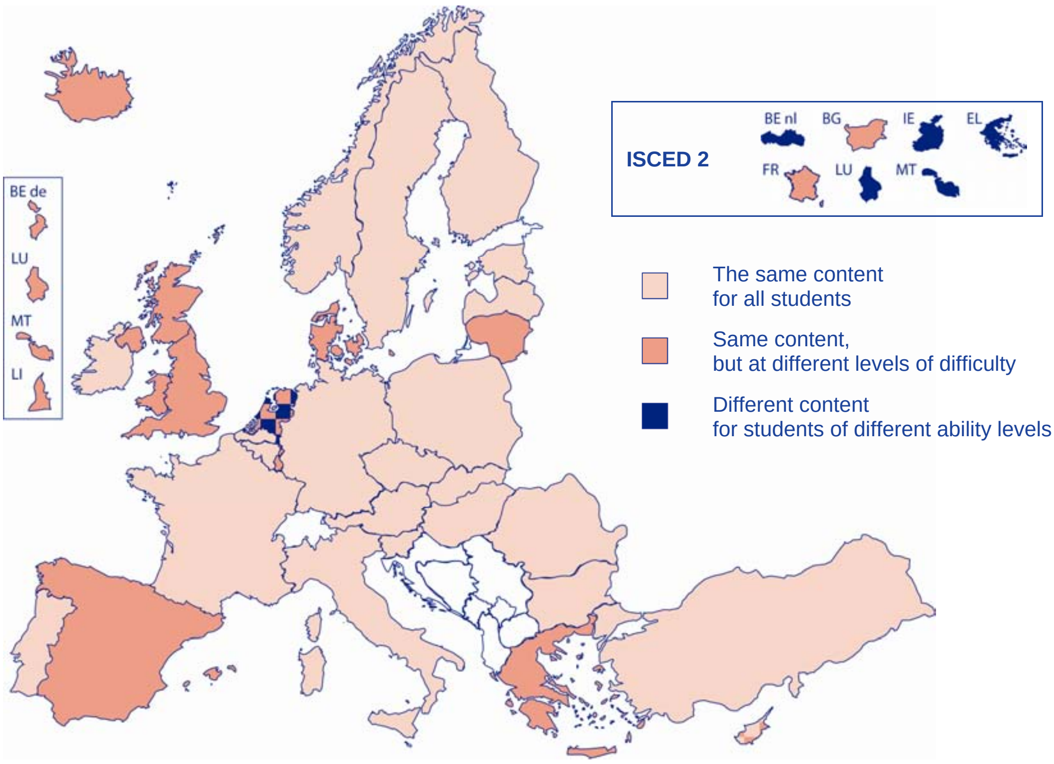
In both contexts, in and outside normal class work, assessment plays an important role that should not be limited to diagnosing potential problems, but should also extend to measuring progress at the end of any period of specific support. The use of a variety of assessment tools is recommended for the accurate identification of individual strengths and weaknesses.

Furthermore, teachers' skills in dealing with students with a range of abilities and interests are essential. A number of countries stipulate that such competences should be acquired during initial teacher training programmes and be further developed through continuing professional development (CPD) activities (see Chapter 6).

Curriculum adaptation

Information from curriculum and other steering documents demonstrates that in half of all European countries mathematical subject content is the same for all students, regardless of ability level (see Figure 4.3). However, differentiated teaching is provided in many countries and is more common at lower secondary level than at primary level. Differentiated teaching usually involves teaching the same content but at different levels of difficulty, a practice that is common in half of the countries. At lower secondary level, students in several countries are taught different subject content.

Figure 4.3: Differentiation of curriculum content according to ability, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

Explanatory note

Information does not include curriculum differentiation that specifically relates to SEN.

In **Spain**, minor curricular adaptations may be made in all subjects at both primary and lower secondary levels for students who don't broadly achieve the general objectives of the stage. The curriculum for these students is adapted to their specific needs; it includes the same objectives and content as for other students, but at a different level of difficulty. Apart from these measures, a specific Curricular Diversification Programme is in operation at lower secondary level. It involves ability-based grouping and a significant modification of the curriculum whereby mathematics and sciences are taught together according to a specific methodology. It is normally a two-year programme for students who have not achieved the general objectives of the third year of lower secondary education or for students who, having finished the second year, are not ready for promotion to the third year and have already repeated a year once.

In **Ireland**, all subjects in lower secondary education including mathematics are offered at two levels: the Higher-Level course in mathematics encompasses the content of the Ordinary-Level course but extends it significantly.

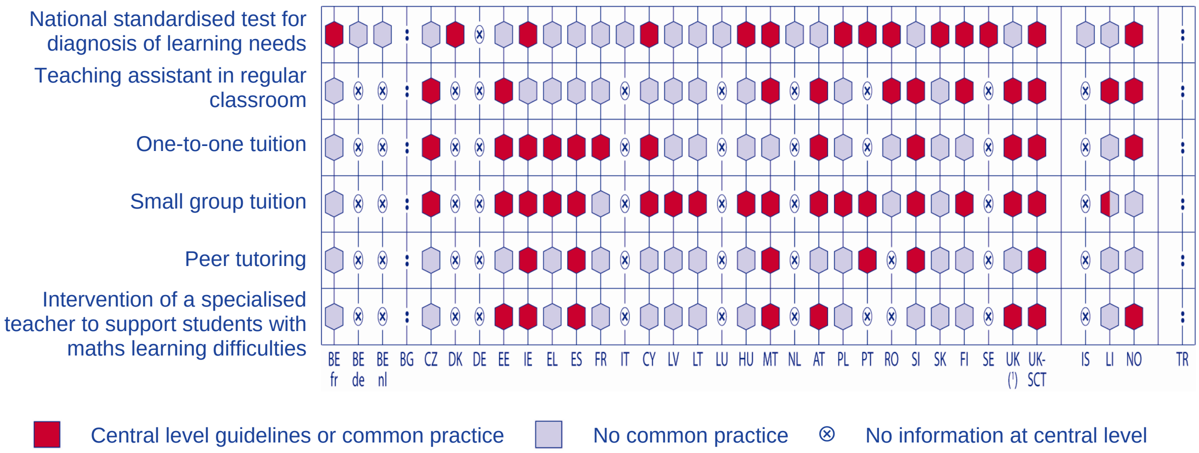
In **Malta**, in the first three years of primary education, less able pupils are being identified and given additional support through the Core Competences project to bring them up to par with their peers. At secondary level, there are four different programmes of study to cater for different levels of ability.

In the **United Kingdom** (**England**, **Wales** and **Northern Ireland**), teachers are expected to differentiate their teaching to meet the needs of students of different levels of ability while following the same programme of study. In line with this expectation, the statutory curriculum separates programme content from attainment targets. Schools have autonomy over the grouping arrangements and, in practice, tend to differentiate groups or classes according to ability at lower secondary level.

In the **United Kingdom** (**Scotland**), there is only one curriculum and it has been specially designed to meet the needs of all students. All students experience the same curriculum but at different levels of challenge and at a different pace of learning. For mathematically-challenged students, there are certain concepts, such as algebraic expressions, that may be approached at a basic level or may well be circumvented. On the other hand, social concepts, such as money, time and measure might be given more attention. Effective teachers will make the best decisions for individual students.

Apart from curriculum modification, several other main approaches and methods are commonly used in addressing underachievement in mathematics (see Figure 4.4). The types of support offered generally include one-to-one and small group tuition, whereas use of a teaching assistant in the regular classroom and intervention by a specialised teacher are much less common. Indeed, specialised teachers who may be either mathematics teachers or teachers who specialise in learning difficulties generally intervene only in Estonia, Ireland, Spain, Malta, Austria, the United Kingdom and Norway.

Figure 4.4: Central level guidelines and common practices for supporting low achievers in mathematics, ISCED levels 1 and 2, 2010/11



Source: Eurydice.

Country specific note

Czech Republic: Support measures are provided for students with special education needs which also include socially-disadvantaged students.

It should be noted that central education authorities only rarely provide specific guidelines in this area. Such guidelines are in operation for instance in Ireland, Spain, Malta and Slovenia. More often, the choice of methods and the way support measures are implemented are decided at school level and/or by individual teachers. In some countries, this high level of autonomy is accompanied by information gathering that enables central authorities to have an overview of which approaches are commonly practiced (United Kingdom and Norway) or not (Lithuania and Poland). In other countries, such as Germany, the Netherlands, Portugal, Sweden and Iceland, no aggregated statistics on commonly used approaches are available at national level.

Diagnostic tools

A number of countries note that an important objective of their policies at primary level is to identify students that need additional support in mathematics. This is done by using a range of assessment tools. In Ireland, for instance, these tools include teacher observation, work analysis, screening tests, standardised test results and diagnostic test results.

In some cases, the identification of students with learning difficulties is left entirely to the class teacher or, more often, it is based on a combination of teachers' marks and the results of nationally standardised tests. Portugal falls into the first category with teachers responsible for analysing student performance, identifying students with potential problems, diagnosing their learning difficulties and writing reports on students suggesting ways to improve performance. These reports are discussed at school level where a decision on the required remedial actions is made.

In other countries, some diagnostic tools are centralised: national tests to identify individual learning needs are set in Cyprus at the end of year 6 or at the beginning of year 7; in Bulgaria at the end of years 4, 5, 6; and in Sweden in years 3 and 6. In Norway, compulsory diagnostic tests in numeracy and arithmetic skills are organised at year 2. These are supplemented by voluntary tests in numeracy and arithmetic skills at years 1 and 3. In addition, Norwegian teachers are encouraged to use web-based diagnostic tests ([12]).

One-to-one and small group tuition

Several countries report the use of one-to-one tuition.

> In **France**, at primary level, the Ministry has prescribed two hours of personalised work per week, which can be used for remedial work with students in classes CE1 and CM2 who have underperformed in national testing in mathematics. In **Greece**, also at primary level, students can have up to six hours per week of individual work. In **Romania**, this approach is mainly used in recovery programmes in rural schools.

Another common approach is tuition in small groups which in Bulgaria, Greece and Lithuania takes place for up to 2 hours per week at the end of the normal school day.

> In **Spain**, students in the last two years of primary and the first three years of secondary education receive support in groups of 5-10, outside school hours, for up to four hours per week. This supplementary tuition is delivered either by university students or by regular teachers.

> In **Ireland**, additional teaching is delivered by learning support teachers; students are usually withdrawn from their normal classes and taught within small groups, although there is growing emphasis on the provision of support to the target students within classrooms. Schools are advised that the duration of the support should cover a school term of 13 to 20 weeks, and not exceed two to three years.

> In **Slovenia**, individual or small group assistance is provided within normal classes or at the end of the school day; teaching assistance is provided by mathematics teachers with additional professional knowledge or by specialist teachers (specialist and remedial pedagogues).

([12]) KIM (Quality in Mathematics education): http://www.tfn.no

In the **United Kingdom** (**England**), the focus of the Every Child Counts programme is on the lowest attaining pupils in Year 2 of primary education. It aims to enable them to reach the expected levels of attainment at Key Stage 1 and beyond. The programme provides training and support for teachers so they can work with children in one-to-one and/or small group intervention sessions. Pupils receive daily intervention sessions for approximately twelve weeks ([13]).

Common implementation problems

The organisation and implementation of measures to tackle low achievement can be affected by a number of obstacles including inadequate resources, lack of appropriate diagnostic tools, difficulties in selecting subject topics for intervention and insufficient teacher qualifications and skills.

Another important constraint can be the lack of sufficient evidence on the advantages and effectiveness of specific forms of support. No firm evidence on the impact of factors such as duration, starting time, intensity, type of assessment and qualifications, and type of the teaching staff involved is available. There is also a need for longitudinal studies that assess the long-term benefits of interventions (Williams, 2008; Dowker, 2009).

Summary

As demonstrated in this overview, in the majority of European countries, central education authorities prescribe or recommend measures, or give assistance to teachers and schools to address low achievement in mathematics. Central level measures range from compulsory, comprehensive national programmes to support for a limited number of activities such as teacher training courses, research projects or data banks of mathematics learning resources. In some countries, in line with the high degree of decentralisation of the school system and teaching autonomy, the design and implementation of measures to tackle low achievement are left entirely to the discretion of teachers, schools and school providers.

According to research, to be effective, the measures taken to address low achievement should be embedded in curriculum content, classroom practices and teacher education and training. Some measures are applicable to all students in the classroom and include teaching methods such as differentiated learning and contextualisation which help raise student performance and motivation overall. Others focus on low-attaining students and encourage prevention, early diagnosis and individual interventions. Teachers who are specialised in mathematics learning difficulties or assistants who can help classroom teachers to support low-achieving students are available in only a few countries.

Overall, it appears that there is a marked need to collect and systematically use robust evidence on effective intervention and support. Another important finding of the analysis of national information is the need to improve the monitoring and evaluation of measures to address low achievement as only a handful of countries have recently conducted evaluations of the impact of support programmes. Few countries have established national targets to reduce the numbers of low achievers in mathematics.

([13]) http://www.everychildachancetrust.org/smartweb/every-child-counts/introduction. See also http://www.edgehill.ac.uk/everychildcounts

CHAPTER 5: IMPROVING STUDENT MOTIVATION

Introduction

At school, and also in wider society, mathematics is sometimes perceived as a difficult and abstract subject which involves learning a lot of processes and formulae that not only appear to be unconnected with each other but also seem irrelevant to students' lives. Negative attitudes towards mathematics and a lack of confidence in 'being good at it' can affect achievement and determine whether students choose to study mathematics beyond compulsory schooling. Schools and teachers can play an important role in increasing student interest and engagement, and in making mathematics teaching more meaningful.

Improving students' motivation to learn mathematics is crucial for a number of different reasons. At the EU level, the Education and Training 2020 strategy underlines the importance of providing efficient and equitable education of high quality in order to improve employability and allow Europe to retain a strong global position. In order to achieve this objective, continued attention must be paid to raising the level of basic skills such as literacy and numeracy (Council of the European Union, 2009). Another rationale for strengthening the motivation to learn mathematics relates to the more immediate policy concern of skills shortages in the labour market. Young people's interest in mathematics and related subjects is important therefore as it is a strong determinant of career choices in mathematics, science and technology (MST) related fields. Furthermore, maintaining high-level skills in these fields is crucial to the economy and so aiming for a high proportion of MST graduates continues to be an important objective in all European countries.

This chapter provides an overview of the policies and initiatives which are intended to increase students' motivation to learn mathematics. Section 1 reviews the main outcomes of international and national research and surveys. Sections 2 and 3 present national strategies and practices for encouraging students to learn mathematics and for fostering positive attitudes towards MST-related subjects, in general, and mathematics in particular. Finally, section 4 highlights the policy concerns related to the take-up of mathematics in higher education and skills shortages in the labour market. The issue of gender differences is addressed throughout the whole chapter; this has been the focus of attention not only in the research field of motivation in mathematics but also of policy measures related to participation in higher education.

5.1. Providing a theoretical and evidence-based framework

Students bring a set of personal attitudes to school that have a marked effect on their achievement. However, these attitudes can be influenced by the teaching and learning that takes place in school. Over the last decades, educational research has thoroughly investigated the concept of motivation and highlighted its effects on learning in school. All students must be motivated in some way to engage in school activities, including in mathematics learning, and the nature of that motivation largely determines the outcome of their efforts.

Although the term 'motivation' is commonly used, there are many definitions in a variety of contexts. In the context of education, learner motivation may be defined as 'a range of an individual's behaviours in terms of the way they personally initiate things, determine the way things are done, do something with intensity and show perseverance to see something through to an end' (Lord et al. 2005, p. 4).

The academic literature distinguishes between two motivational concepts – intrinsic and extrinsic motivation (Deci and Ryan, 1985). Extrinsically motivated students engage in mathematical activities in order to attain external rewards, such as praise from teachers, parents and peers, or to avoid punishment or negative feedback. Students who are intrinsically motivated, on the other hand, learn mathematics for their own interest, enjoyment and pursuit of knowledge (Middleton & Spanias, 1999). Students who are intrinsically motivated, therefore, focus on understanding concepts. Consequently, intrinsic, rather than extrinsic, motivation benefits students in the process and results of mathematical activities (Mueller et al., 2011).

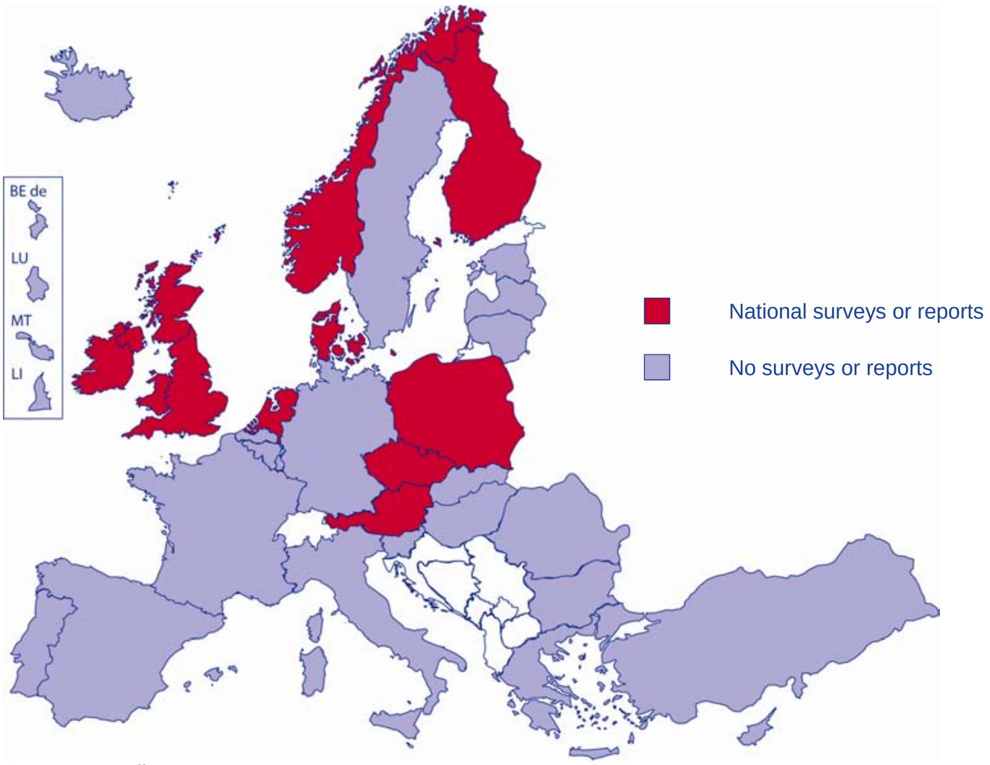
Intrinsic motivation leads to self-efficacy, i.e. an individual's beliefs about their own abilities. According to Bandura (1986), students' self-efficacy beliefs often predict their ability to succeed in a particular situation. Studies suggest that particularly in mathematics self-efficacy is a clear predictor of students' academic performance (Mousoulides & Philippou, 2005), and that students with highly developed self-efficacy beliefs use cognitive and meta-cognitive learning strategies more effectively while being more aware of their own motivational beliefs (Mousoulides & Philippou, 2005; Pintrich, 1999).

Thus, student motivation is in itself related to a range of concepts:

- self-concept, i.e. how individuals perceive themselves, in this instance as learners, including their sense of self-efficacy;
- self-regulation, including the capacity to develop learning strategies and resilience;
- learner involvement, engagement and participation;
- attitudes towards education and learning;
- impacts on the learner such as on their self-esteem or through stress and anxiety

(Lord et al., 2005).

Figure 5.1: National surveys and reports on motivation in mathematics, 2010/11



Source: Eurydice.

Although this chapter refers to the general term 'motivation', international surveys such as PISA and TIMSS use concepts such as 'students' beliefs' and 'students' attitudes'. PISA 2003, focusing on mathematics, examined students' beliefs about mathematics which were defined as 'self-concept' and 'self-efficacy'. TIMSS examined students' attitudes towards mathematics, the value they placed on mathematics for their education and future work and their confidence in their mathematics ability.

Besides international surveys, some national surveys or reports examine factors related to motivation in mathematics. As shown in Figure 5.1, national surveys and reports on motivation in mathematics have been conducted in nine countries, the Czech Republic, Denmark, Ireland, the Netherlands, Austria, Poland, Finland, the United Kingdom and Norway. Most often, these reports examine the relationship between motivation and achievement, students' perceptions of mathematics, innovative teaching methods for increasing engagement and tackling gender differences. Some of the outcomes, which are mostly in line with the key findings of research and international studies, are presented in more detail below.

Motivation and achievement

It is generally assumed that children learn more effectively when they are interested in what they learn. Moreover, they may achieve more if they enjoy what they learn. The academic literature has indeed shown that motivation is an important factor to consider in the context of academic achievement (e.g., Grolnick et al., 1991; Ma & Kishor, 1997). Studies have indicated, for example, that intrinsic motivation positively influences academic achievement (Deci & Ryan, 2002; Urdan & Turner, 2005).

In the context of mathematics learning, therefore, it appears that students who enjoy the subject increase their intrinsic motivation to learn, and vice-versa (Nicolaidou & Philippou, 2003). When students are motivated to learn mathematics, they spend more time on mathematical tasks and tend to be more persistent in solving mathematical problems (Lepper & Henderlong, 2000). They may also be more open to taking a larger number of mathematics courses and to pursuing a career related to mathematics (Stevens et al., 2004). As a result, students' motivation has an important impact on their mathematics achievement.

The TIMSS international survey has also investigated the link between motivation and achievement in mathematics and has revealed that, in general, positive attitudes seemed to be related to higher achievement at both fourth and eighth grades. The relationship between attitudes and achievement seems to be stronger at the eighth grade. In 2007, on average in the participating EU countries ([1]), at the fourth grade, those students who had very positive attitudes reached a score 20 points higher than those who had negative attitudes. At the eighth grade, the difference was 42 points (for country data, see Mullis et al. 2008, pp. 175-177).

Some national surveys have likewise examined this topic. The Czech survey 'Magma' ([2]) found that in the ninth grade classes, where most students felt satisfied with their performance in mathematics lessons, the results were twice as good as in other classes. However, students in the same class with either low or high achievement often answered similarly so this might be linked to the qualities of the teacher.

([1]) Here and elswhere, the Eurydice calculated EU average refers only to the EU-27 countries which participated in the survey. It is a weighted average where the contribution of a country is proportional to its size. When making comparisons between fourth and eighth grades, it is important to take into account that different EU-27 countries participate in the assessments (see 'Achievement in mathematics: evidence from international surveys').

([2]) http://www.novamaturita.cz/magma-1404033815.html

Students' motivation and achievement in mathematics may also be influenced by the importance they attach to this subject. The TIMSS survey gathered information about whether eighth grade students perceive mathematics achievement to be advantageous for their future education and career. In 2007, 68 % of students, on average in the EU, placed a high value on mathematics. Only 6 % of eighth grade students did not perceive mathematics as useful in their future education or work. The highest percentage of students who perceived mathematics achievement as advantageous to their future career was in Lithuania and Turkey with 85-87 %. In Italy, eighth grade students valued mathematics less than students in other participating EU countries with approximately every second student valuing mathematics highly (Mullis et al. 2008, p. 179). On average in the participating EU countries, eighth grade mathematics achievement was 31 points higher among students who valued mathematics highly than among those who did not.

It should be noted, however, that motivation for doing mathematics is not a stable learner characteristic but a dynamic, changeable feature. For example, the thematic report of the Czech School Inspectorate (2008) and the 2008 Scottish Survey of Achievement ([3]) compared the motivation of students in different school years. Both reports concluded that student motivation declines throughout secondary school – a finding which highlights the important role of teachers and the teaching process in using diverse teaching methods and supporting learner motivation.

TIMSS results also confirm that fourth grade students had much more positive attitudes towards mathematics than eighth grade students. On average, in the participating EU countries, 67 % of fourth grade students and only 39 % of eighth grade students had very positive attitudes towards mathematics ([4]). However, it is important to take into account that different sets of EU countries assessed fourth and eighth grade students. Seventy per cent or more of fourth grade students had very positive attitudes in Germany, Italy, Lithuania and Slovenia. At the eighth grade, only Turkish students had similarly positive attitudes. In contrast, in Slovenia, eighth grade students had the least positive attitudes towards mathematics (more than 50 % were negative towards mathematics) (Mullis et al. 2008, pp. 175-177).

The impact of student attitudes, beliefs and self-confidence

An important aspect related to motivation and achievement is the impact of students' attitudes towards mathematics. Attitudes are psychological states made up of three components: a cognitive component, an emotional component and a behavioural component. In the context of education, they are seen as one of the personal factors that affect learning (Newbill, 2005).

Research in mathematics education has highlighted that attitudes play a crucial role in learning the subject (Zan & Martino, 2007). Moreover, students' positive attitudes towards mathematics, which may be enhanced through effective teaching strategies, can promote learning achievement (Akinsola & Olowojaiye, 2008). Negative feelings or anxiety, on the other hand, can become a barrier to achieving good learning outcomes. Mathematics anxiety is thus an affective, or emotional, state, which has been shown to impair student performance (Zientek & Thompson, 2010; Zientek et al., 2010).

Another variable related to attitudes which affects motivation is self-belief. Students' belief in their own abilities can play an important role in mathematics performance and achievement (e.g., Hackett & Betz, 1989; Pajares & Graham, 1999; Pajares & Kranzler, 1995). According to Hattie's (2009) synthesis of more than 800 meta-analyses relating to achievement, students' beliefs determine their

([3]) http://www.scotland.gov.uk/News/Releases/2009/03/31134016
([4]) TIMSS Index of Students Positive Affect Toward Mathematics

personal responsibility for their learning. The idea that higher achievement is a direct result of one's efforts and interest is critical to success.

One particular motivational belief for student achievement is self-efficacy. In the context of mathematics, research evidence shows that self-efficacy, measured as the level of student confidence, can predict mathematics performance (Pajares & Miller, 1994; Pajares & Kranzler, 1995; Pajares & Graham, 1999).

Similarly, TIMSS results indicate that students' confidence about their abilities in mathematics ([5]) is related to their achievement in mathematics at both fourth and eighth grades. In 2007, on average in the participating EU countries, at the fourth grade, students who expressed considerable self-confidence scored 74 points higher than those who had low levels of self-confidence in their mathematical abilities. At the eighth grade, the difference was 88 points.

Yet it is important to note that students' confidence in learning mathematics at the eighth grade was lower (on average in participating EU countries 47 % of students had high self-confidence) than at the fourth grade (67 %). At the fourth grade, the highest levels of self-confidence were reported in Denmark, Germany, Austria, and Sweden, with 70 % or more students having considerable self-confidence, and the lowest levels in the Czech Republic, Latvia, Lithuania and Slovakia, with less than 60 % of students being confident in their abilities in mathematics (Mullis et al. 2008, p. 182). At the eighth grade, self-confidence levels were highest in Cyprus, the United Kingdom (England and Scotland) and Norway (50 % or more at the high level) and lowest in Bulgaria, Malta, Romania and Turkey (less than 40 % at the high level) (ibid., p. 183).

In order to address these affective issues related to mathematics, the Finnish study 'LUMA-Finnish Success Now and in the Future – Mathematics and Science Advisory Board Memorandum' ([6]) suggests that positive attitudes towards MST should be promoted among children as early as in pre-primary education. In particular, students with learning difficulties should be identified at an early stage, as unsolved problems can lead to frustrations and anxiety towards mathematics. The latter highlights the role of teachers in applying appropriate and timely teaching methods. Some other reports point to the importance of parental involvement in the learning process. The 2006 report of the Danish Evaluation Institute stresses the need to strengthen home-school collaboration so that parents increasingly become able to support the school's work in promoting their children's positive attitude towards mathematics. The STEM (science, technology, engineering and mathematics) Careers Awareness Timeline Pilot ([7]) in the United Kingdom concludes that parents can play a significant role in influencing young people's choices about careers in the field.

Finally, some surveys (e.g. 'BètaMentality 2011-2016' ([8]) in the Netherlands, 'Lily' ([9]) and 'ROSE' ([10]) in Norway) focus on tertiary students' perceptions of MST subjects. These provide valuable information that can be used by primary and secondary schools in adapting their teaching methods and making subjects more attractive to students. This, in turn, is important for the recruitment of MST students to tertiary education.

([5]) TIMSS Index of Students’ Self-Confidence in Learning Mathematics.
([6]) http://www.oph.fi/instancedata/prime_product_julkaisu/oph/embeds/110468_luma_neuvottelukunnan_muistio_2009.pdf
([7]) http://www.nationalstemcentre.org.uk/res/documents/page/lengthening_ladders_shortening_snakes.pdf
([8]) http://www.platformbetatechniek.nl//docs/Beleidsdocumenten/betamentality20112016engels.pdf
([9]) http://www.naturfagsenteret.no/c1515601/prosjekt/vis.html?tid=1519408
([10]) http://www.uv.uio.no/ils/english/research/projects/rose/

Teaching methods to increase student motivation

Mathematics teaching at school should encourage students' motivation to actively participate in the learning process. The nature of the tasks and exercises used for instruction have a great influence on whether students feel challenged and interested in mathematics and therefore motivated to engage with the learning process.

Research on the key influences on students' positive attitudes towards mathematics suggests that teaching methods and tasks must be engaging, diversified and connected to students' everyday life. In this way, students who are involved in the learning process will acquire knowledge that is relevant for their lives (Piht & Eisenschmidt, 2008). To develop intrinsic motivation, mathematics teaching and learning must take place in a supportive learning environment where students are encouraged to communicate their understanding of the tasks and where their ideas are valued and appreciated. Such an environment supports students' self-concept, their self-efficacy and their enjoyment of mathematics as they discuss and share their understanding with their peers (Mueller et al., 2011). These teaching approaches, therefore, establish the necessary conditions for enhancing student motivation as well as achievement in mathematics.

National surveys and reports also cover issues related to mathematics teaching approaches and their impact on student motivation. These aspects are analysed in more detail in Chapters 2 and 6. However, two examples of national surveys and reports linked to motivation can be mentioned here. The thematic report of the Czech School Inspectorate (2008) contained, amongst other things, an assessment of teachers' abilities to impact on students' motivation towards numeracy. The United Kingdom, in its Careers Awareness Timeline Pilot (2009), concluded that continuing professional development is central to improving teachers' awareness of the relationship between the quality of subject teaching, enjoyment of learning and subject choice as well as their knowledge about STEM (science, technology, engineering and mathematics) careers.

Other reports highlight a need to increase the variety of innovative teaching methods (Denmark) which attract students' attention and engage them more in learning process (United Kingdom). Practical and interesting exercises close to students' everyday life, making use of their experience from other subjects and linking them with mathematics (Czech Republic), as well as promoting a creative attitude and collaborative approach are suggested to overcome negative attitudes of students who find mathematics challenging and uninspiring (United Kingdom (Scotland)).

Gender differences in motivation and achievement

The gender dimension is a recurring element of research in the field of mathematics education. Although the stereotypical view is that girls and women lack mathematical ability, an increasing amount of research provides evidence that males and females differ very little in their mathematics achievement (e.g. Hyde et al., 1990; Hyde et al., 2008; Else-Quest et al., 2010).

However, studies show that girls tend to report less positive attitudes and confidence in their mathematics ability, and that the gap widens throughout schooling when boys report greater self-confidence (Hyde et al., 1990; Pajares & Graham, 1999). Girls have also been found to have higher levels of mathematics anxiety and lower self-belief (Casey et al., 1997; McGraw et al., 2006). This can have important implications, as evidence shows, because teachers tend to associate students' confidence with ability. As a result, they may underestimate girls' mathematical abilities as they are more likely to demonstrate more mathematics anxiety than boys even if they have high ability (Kyriacou & Goulding, 2006).

The PISA 2003 survey confirmed that although female students do not usually perform at much lower levels than males, in almost all countries they tend to report lower levels of mathematics-related self-efficacy. Similar results emerge for students' self-belief in mathematics, where males tend to have a more positive view of their abilities than do females in most countries. Finally, on average, females experience significantly more feelings of helplessness, anxiety and stress in mathematics classes than males. There were statistically significantly higher levels of anxiety among females in Denmark, Germany, Spain, France, Luxembourg, the Netherlands, Austria, Finland, Liechtenstein and Norway (OECD 2004, p. 155).

TIMSS 2007 data also shows that on average, in the participating EU countries, girls had lower self-confidence in their abilities in mathematics than boys. At the fourth grade, 61 % of girls and 71 % of boys expressed considerable self-confidence in their abilities in mathematics, while 11 % of girls and 7 % of boys were not confident in their mathematics ability. Only in Sweden, the United Kingdom (Scotland) and Norway did the proportions of girls and boys with high self-confidence in their mathematics abilities not differ. At the eighth grade, 42 % of girls and 52 % of boys rated their abilities in mathematics as high, while 24 % of girls and 17 % of boys were not confident in their abilities. The proportion of boys and girls with high self-confidence in their mathematical abilities were similar in Bulgaria, Lithuania, Romania and Turkey (Mullis et al. 2008, pp. 184-185).

Both surveys, therefore, reported similar findings about students' attitudes towards mathematics. However, the most important finding seems to be that the gender gap is wider with respect to attitudes towards mathematics than in actual levels of mathematics achievement.

National surveys reflect similar gender differences with regard to attitudes, beliefs in own abilities and participation of boys and girls in further studies in mathematics. The Finnish study 'LUMA-Finnish Success Now and in the Future – Mathematics and Science Advisory Board Memorandum' reports that the difference between the self-confidence of boys and girls in mathematics is large, although knowledge-based differences are not statistically significant. The study concludes that the participation of female students in MST-related subjects must be supported and their self-confidence in mathematics encouraged.

Overall, recent analyses all point to the importance of increasing motivation at school, particularly among female students. The use of appropriate teaching methods can help motivating students to learn mathematics, developing a deeper interest in this field and remaining engaged and interested throughout primary and secondary levels. This has a crucial impact not only on school achievements, but also on influencing their choice of future field of study and career.

5.2. National strategies to improve student motivation in mathematics learning

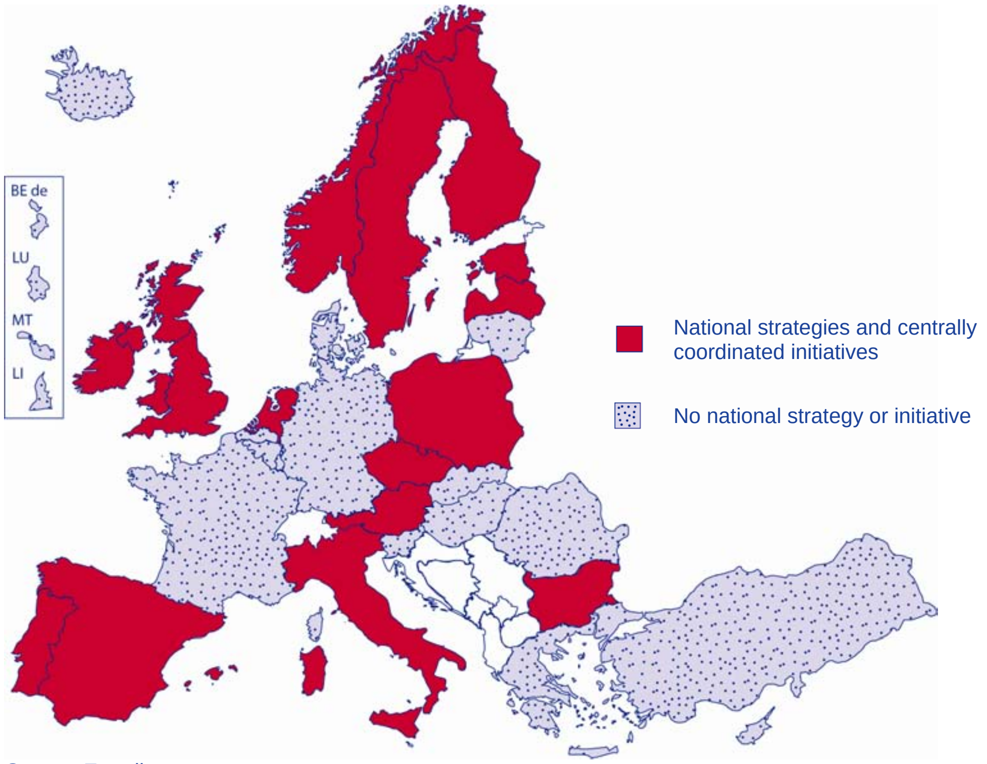
Based on the outcomes of international and national surveys, European countries have started to adopt national strategies and initiatives to improve student motivation in mathematics education. Hence, apart from developing new teaching approaches, reviewing curricula and adapting teacher education (see Chapters 1, 2 and 6), raising levels of motivation has come to be seen as a key element in improving mathematics performance.

At present, less than half of the European countries have national strategies or centrally coordinated initiatives that, among others things, aim to increase motivation in mathematics learning (see Figure 5.2). These often form part of a broader policy for promoting the learning and teaching of mathematics, science and technology (for strategies and policies with respect to the promotion of science education, see EACEA/Eurydice, 2011c).

Examples of some of the current national strategies or centrally coordinated initiatives focusing on increased motivation in mathematics learning are outlined below:

Finland set up an institutional framework for promoting the learning, studying and teaching of mathematics, science and technology. The 'LUMA Centre' ([11]) is an umbrella organisation for cooperation between schools, universities, business and industry, coordinated by the Faculty of Science of the University of Helsinki. Its main objective is to support and promote teaching and learning of MST at all levels. The centre develops activities for pupils, such as MST camps, as well as providing in-service training and workshops for teachers. In addition, LUMA serves as a resource centre for mathematics supplying various teaching and learning materials.

Figure 5.2: National strategies to increase student motivation in mathematics learning, 2010/11



Source: Eurydice.

Explanatory note

The figure refers to documents adopted by national authorities as well as programmes or projects officially recognised/coordinated by national authorities. Mathematics Olympiads and other competitions are not included, but listed among activities in section 5.3.

Austria launched the nation-wide project 'IMST' *(Innovationen machen Schulen Top)* ([12]). It aims at improving instruction in mathematics, science, IT and related subjects. The focus is on students' and teachers' learning. The project involves about 5 000 teachers across Austria who participate in projects, attend conferences, or cooperate in regional and thematic networks. The IMST-programme, Regional and Thematic Networks, supports regional networks in all nine Austrian provinces, and three thematic networks. Within the IMST-Fund, teachers put into practice innovative instructional projects and receive support in terms of content, organisation and finance. In the programme 'Examination Culture', teachers reflect on the different forms of assessment in a range of seminars. Gender sensitivity and gender mainstreaming are important principles of the project, and their implementation is supported by the Gender Network. In order to investigate the impact of IMST, evaluation and research is integrated at all levels. An evaluation study shows

([11]) http://www.helsinki.fi/luma/english/index.shtml
([12]) http://imst.uni-klu.ac.at/

that students who were involved in the IMST-programme revealed high levels of intrinsic motivation as well as interest in the subject and positive self-esteem (Andreitz et al., 2007).

Thus, the initiatives in Austria and Finland target a wide range of students throughout the whole education system – in Austria recent initiatives also focus on the kindergarten and, similarly, in Finland on pre-primary education. On the other hand, in Ireland, Spain and Portugal, comprehensive action plans focus on compulsory education. They all aim at improving motivation and encouraging positive attitudes towards mathematics learning.

In **Ireland**, the Curriculum reform initiative 'Project Maths' ([13]), led by the National Council for Curriculum and Assessment (NCCA), began in 2008 in an initial group of 24 schools and is being implemented nationally for incoming first year and fifth year students in 2010. It aims to provide for an enhanced student learning experience and greater levels of achievement for all. Much greater emphasis is placed on student understanding of mathematical concepts, with increased use of contexts and applications that enables students to relate mathematics to everyday experience. The initiative also focuses on developing students' problem-solving skills. Assessment reflects the different emphasis on understanding and skills in the teaching and learning of mathematics.

The Ministry of Education in **Spain** published the Action Plan 2010-2011 which targets various subjects including mathematics with the aim of achieving educational success for all pupils at the end of compulsory education. The actions include an amended curriculum for lower secondary schools, personalised learning and parent involvement, which should also bring higher levels of motivation in mathematics. Some of the Action Plan's funds have been transferred to the Autonomous Communities, which are also implementing related policies.

In **Portugal**, the 'Action Plan for Mathematics' has been launched with the aim of improving the teaching and learning of mathematics in compulsory education. The core of the Plan is to support the development of projects designed by schools, which take into consideration the specific context of the school community and their needs. Teachers are seen as the major contributors in the complex process of improving teaching methods and consequently students' learning. Ninety-one per cent of schools are involved in the Action Plan. The projects allow students to dedicate more time to the study of mathematics and focus on exploration, investigation and problem-solving. An important aspect is peer-teaching in the classroom, involving two mathematics teachers or a mathematics teacher and a teacher of another subject. It allows improved dynamics between teachers and a more integrated approach towards mathematics and other subjects. According to the latest evaluation, improvements in student motivation and attitudes towards mathematics have been observed, particularly in the learning of concepts and procedures.

The strategies and initiatives in Italy, the Netherlands and Norway concentrate primarily on upper secondary education and on encouraging students in MST subjects to continue their studies at tertiary level. In the United Kingdom, the overall aim is to increase participation in mathematics and science at tertiary level but STEM activities (see below) are aimed at pupils of all ages, including those in primary schools, as it is recognised that motivation can be maximised by inspiring pupils early in their school career. The initiatives are mainly intended to address skills shortages in areas requiring high levels of mathematical knowledge (see Figure 5.4).

Italy has started the Scientific Degrees Programme for students in the last three years of upper secondary school which is funded by the Ministry of Education. Among its main objectives are increasing the number of students attending science faculties (to study mathematics degrees in particular), engaging students in mathematics and research, and strengthening cooperation between schools and university teachers. In addition, Italy launched the initiative 'Promotion of Excellence' which rewards students of upper secondary schools who attain outstanding results in various competitions, including in the field of mathematics.

([13]) http://www.projectmaths.ie

The 'Platform Bèta Techniek' ([14]) has been commissioned by the government and the education and business sectors in the **Netherlands** to ensure sufficient availability of people who have a background in MST. The main goal of the organisation is to motivate young people at all educational levels to take an interest in mathematics and science, to increase the numbers choosing to study these subjects and to retain them in the MST field. Members of the Platform closely cooperate with various stakeholders in the education system in order to reach its objectives. Participating schools earn grants by introducing successful innovations into their MST education.

The STEM Programme ([15]) adopted for the whole of the **United Kingdom** aims to improve the provision of support for students aged 3-18 in the field of mathematics. It aims, amongst other things, at widening access to the formal science and mathematics curriculum for all. In addition, **Scotland** has specifically designed the 'Curriculum for Excellence' (CfE) ([16]), which aims to drive a methodology in learning and teaching that motivates and inspires. This new curriculum places literacy, numeracy and health and well-being at the core of learning and, as numeracy is defined as a subset of mathematics, the profile of mathematics is raised within the CfE.

Norway has developed the strategy 'Science for the Future'. As many pupils encounter difficulties with their skills and motivation in mathematics, the Ministry of Education and Research established a working group that has been given the task of considering how mathematics can be made more relevant and engaging for pupils at all levels of education. In addition, the National Centre for Recruitment to Science and Technology has initiated the establishment of a national agency to promote MST role models in the form of ambassadors from a variety of educational pathways and professions. Lower and upper secondary schools can book visits from role models and may also visit them at their workplace.

Countries in Central and Eastern Europe do not indicate overarching national strategies. However, some of them coordinate programmes and projects co-financed by European Structural Funds – an instrument specifically mentioned by the Council to improve, among other things, motivation and performance in mathematics (Council of the European Union, 2010). The projects stress innovative teaching methods, intended to engage students by presenting mathematics in an interesting and motivating way, and with a focus on understanding the importance of mathematics in everyday life.

In the **Czech Republic**, a number of projects related to mathematics have been launched some of which focus entirely on science and technology. The project 'EU Money for Schools' targets seven specific areas of which mathematics is one. As the project's key activities aim at developing mathematical literacy, basic schools may choose topics such as innovation and improvement of teaching methods or individualisation of teaching through teacher training to enhance the effectiveness of mathematics teaching.

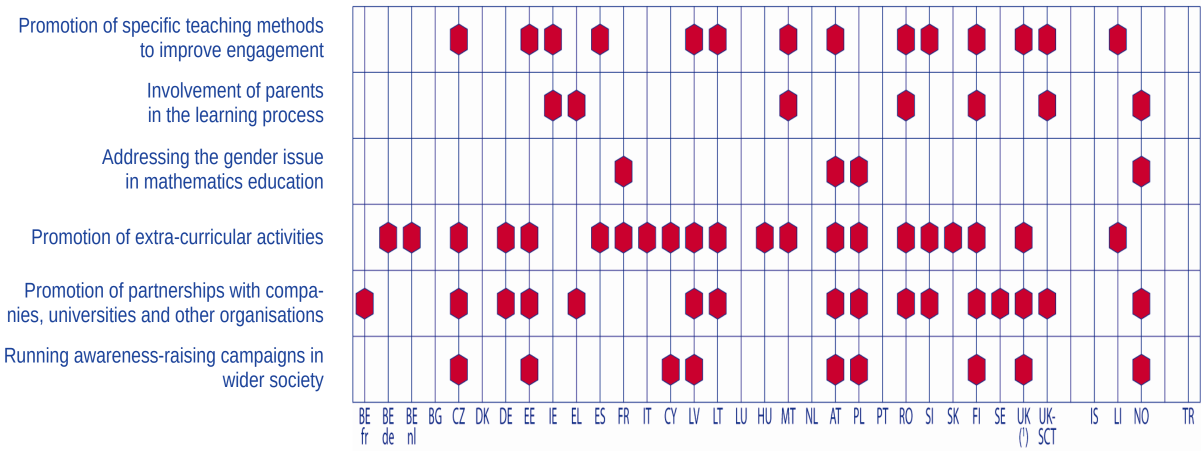
Latvia launched a pilot project 'Science and Mathematics' (2008-2011) with twenty-six schools which is intended to encourage the interest in mathematics of pupils in grades 7-9 and to raise their understanding of the importance of mathematics in everyday life. Among the main activities covered by the project are competitions for pupils published on the project website and the implementation of modified teaching methods. The objective of this project is to identify the most effective teaching methods for motivating students to learn mathematics, such as using active learning, real life examples, didactic games or information technologies. Initial results from an evaluation survey indicate slightly more positive attitudes towards mathematics among students who took part in the pilot project compared to those who did not participate.

([14]) http://www.platformbetatechniek.nl/?pid=49&page=About%20Platform%20Beta%20Techniek
([15]) http://www.stemdirectories.org.uk/about_us/the_national_stem_programme.cfm
([16]) http://www.ltscotland.org.uk/understandingthecurriculum/whatiscurriculumforexcellence/index.asp

5.3. Centrally supported activities to improve attitudes towards learning mathematics

A number of European countries promote activities to encourage positive attitudes towards learning mathematics thereby improving engagement at school and ultimately influencing students' career choices. These activities are mainly implemented within the framework of national strategies and centrally coordinated initiatives. They can be grouped into several topics (see Figure 5.3).

Figure 5.3: Activities supported by central education authorities to improve students' perceptions of mathematics, ISCED levels 1-3, 2010/11



Source: Eurydice.

UK (1) = UK-ENG/WLS/NIR

Country specific note

Ireland: The information refers only to primary education.

Most countries promote one or more activities for improving the perception of and attitudes towards mathematics. Overall, extra-curricular activities are the most common initiatives used to promote mathematics, supported by almost half of all European countries. Fostering partnerships and encouraging specific teaching methods to improve engagement are encouraged by slightly more than one third of countries. Although international and national surveys indicate the need to achieve a gender balance in mathematics learning outcomes, only four countries address this issue through national activities.

Extra-curricular activities

More than half of European countries or regions promote extra-curricular activities that take place outside of normal school time – sometimes during lunch breaks but mostly after school, at weekends or during school holidays. Most of these extra-curricular activities are targeted at talented students. The United Kingdom's STEM programme is an exception as it aims to motivate pupils of all abilities in mathematics and science ([17]).

> In most European countries, mathematics competitions for students are organised at different levels (local, regional and national) and students may also take part in international Olympiads. The mathematical society in **Cyprus**, for example, organises, in cooperation with the Ministry of Education, local and national competitions at all levels of education and encourages students to attend international competitions.

([17]) http://www.stemclubs.net/

Germany promotes federal mathematics competitions ([18]) open to those schools providing course leading to tertiary education. They are organised in three stages over the period of one year.

Competitions at national level intended to motivate pupils to learn mathematics are also well established in **France** and many of them date back to the 1980s. There are twenty competitions organised at regional, county or city level across France.

Schools in some countries promote mathematics outside normal mathematics classes. In a number of countries, gifted students are encouraged to attend mathematics summer schools combining recreation and learning.

Some schools in **Estonia** offer special summer courses for the highest achievers in mathematics. Secondary schools in **Liechtenstein** dedicate two weeks per year fostering peer-learning and activity-based learning as well as applying knowledge in real life contexts via various projects, including in mathematics. One example is the 'Einstein week'.

In **Spain**, gifted students are encouraged to attend a program called EsTalMat (Mathematical Talent Encouragement Programme) ([19]). Launched by the Science Royal Academy and the National Council of Scientific Research (CSIC), the programme has been implemented in several Autonomous Communities. Its goal is to identify, advise and foster, over a two-year period, mathematical talent in 12-13 years old students. It involves 3-hour weekly meetings and activities such as seminars and camps.

Partnerships

Educational institutions often work together with other stakeholders to carry out or improve their activities via partnerships. In a review of effective collaboration involving schools and colleges of education and other organisations, views on the importance of collaboration have been collected and the factors contributing to effective collaboration have been identified (Russell and Flynn, 2000). One of the main reasons for collaboration is to provide 'an improved mechanism to achieve common purposes more readily (i.e. more efficiently, at reduced cost, with better quality) through partnership than alone' (Ibid, p. 200). At the European Union level, the first EU Thematic Forum on School-Business Cooperation ([20]) underlined the many different advantages that collaboration can offer to both schools and business organisations, including increasing interest in maths, science and technology and improving students' motivation to learn and take initiative to create their own learning pathways.

The following examples of partnerships describe activities related to mathematics. However, they often take place in a broader context related to MST partnerships. The Eurydice study on 'Science Education in Europe: National Policies, Practices and Research (EACEA/Eurydice, 2011c) provides further details on science and technology related activities. With respect to mathematics-related activities, sixteen European countries or regions report promoting partnerships between schools and companies, universities or other organisations:

The LUMA Centre in **Finland**, already mentioned above, is an umbrella organisation specifically developed for fostering cooperation between schools, universities, business and industry on mathematics teaching and learning. The Centre also cooperates with government agencies, NGOs, associations, science centres and textbook publishers. In **Sweden**, twenty higher education institutions signed an agreement with the National Agency for Education to work as regional development centres for mathematics. In **Estonia**, the University of Tartu and nineteen partner schools have concluded an agreement to cooperate in various fields including mathematics teaching at lower secondary level.

([18]) http://www.bundeswettbewerb-mathematik.de/
([19]) http://estalmat.org
([20]) http://ec.europa.eu/education/school-education/doc2279_en.htm

The **Latvian** project 'Science and Mathematics' ([21]), also mentioned above, provide support for schools and entrepreneurs in organising activities and competitions to promote pupils' interest in mathematics. The project team organised an interactive exhibition and activities in schools, which is also open to the public, with the objective of changing the perceptions of mathematics among 7-12 grade pupils as well as those of parents and society in general.

The **United Kingdom** launched the STEMNET – Science, Technology, Engineering and Mathematics Network ([22]) – which encourages young people to understand STEM subjects and broaden their opportunities while supporting the country's future competitiveness. The Network involves schools, colleges, business, other organisations and individuals such as local experts. Over 24 000 volunteers participate in the STEM Ambassadors Programme including employers.

Partnerships with companies, universities and other organisations are also a crucial element within the 'Curriculum for Excellence' in the **United Kingdom** (**Scotland**). A major initiative to support the rationalisation of the curriculum is embedding financial education within numeracy. The work done within financial education has resulted in strong links between education and various bodies from the financial sector. Programmes already exist where staff from the financial sector visit schools and work with pupils on key aspects of money management. There are also strong links with the education sector and universities. The mathematics departments in many universities promote mathematics through visits, Saturday programmes and national mathematics challenges. In addition, there are links between education and the voluntary sector.

Specific teaching methods for improving engagement

Apart from extra-curricular activities and partnerships, around one third of countries promote specific teaching methods for increasing engagement (see also Chapter 2). They mainly focus on the use of innovative teaching methods including through the use of ICT. The Eurydice report *Key Data on Learning and Innovation through ICT at School in Europe 2011* examines mathematics teaching and concludes that although ICT use by teachers and students is widely recommended at central level, a large implementation gap remains (EACEA/Eurydice, 2011a). ICT can be effectively used to support teaching and it should offer an opportunity for more interaction and discussion not less (The Royal Society, 2010). More generally, the Council concluded that in order to improve engagement 'learning methods should better exploit children's natural curiosity in mathematics and science from an early age' (Council of the European Union, 2010).

The following country examples provide an insight into a number of specific teaching methods:

The project 'Metodika II' in the **Czech Republic** operates an online portal for teaching methodology ([23]). It promotes the development of a community where teachers can share their experiences of effective teaching methods in order to improve the quality of education. The portal has several sections including one on mathematics teaching, and it offers articles, digital learning materials and e-learning courses.

Romania puts an emphasis on active participatory methods and active learning using cooperative strategies (in pairs or in groups). In other words, it recommends a shift from teaching from the front to cooperative teaching and learning in order to improve motivation and engagement in mathematics.

As part of the support for primary schools in **Ireland** participating in the DEIS (Delivering Equality of Opportunity in Irish Schools) educational inclusion programme, the Department of Education and Skills is rolling out the intensive Maths intervention programme 'Maths Recovery' ([24]) as one of the key actions to improve engagement and numeracy outcomes in primary schools situated in disadvantaged areas. This scheme involves the training of Maths Recovery specialists and classroom teachers in the principles and practices of Maths Recovery.

([21]) http://www.dzm.lv/skoleniem/pasakumi/; http://www.dzm.lv/par_projektu/dabaszinatnu_un_matematikas_nedela_2011
([22]) http://www.stemnet.org.uk/
([23]) http://www.rvp.cz
([24]) https://sites.google.com/a/pdst.ie/pdst/maths-recovery

General promotion campaigns

Only nine countries or regions undertake campaigns for promoting mathematics in the wider population. Examples of such campaigns include:

Poland launched a promotional campaign 'Math – look how easy it is' consisting of a series of different TV spots with two components: short TV spots broadcasted at prime time, featuring celebrities as well as different professionals (sailors, pole-jumpers, photographers, etc.), which showed the value of mathematics in everyday situations and in their own work in particular; short TV broadcasts directed at lower and upper secondary students focused on interesting everyday-life mathematical problems (e.g. how to decide which banks offer the best value) and related exercises.

In the **Czech Republic**, the 'Support for Technology and Science Fields' project (2009-2011) is a popularization project aimed at introducing a system of marketing support for the fields of science and technology at universities and other higher education institutions. Project activities are divided into three major pillars: motivation activities, communication, and teaching support which are all directly and indirectly aimed at potential students. The project is a response to the continuing shortages of graduates from universities and other science and technology higher education institutions.

The **Norwegian** survey 'Lily' *(Vilje-con-valg)* ([25]), while aiming to contribute to improving recruitment, retention and gender equity in science, technology, engineering and mathematics (STEM) careers, revealed that there were few visitors to campaign websites established by STEM businesses and other professional organisations. Furthermore, commercials for higher educational institutions had less impact on student choices than visiting the institutions themselves.

Involvement of parents

Parental involvement and encouragement from an early age can have a significant impact on students' mathematics learning. Individual, family and early home learning environment factors are important predictors of children's cognitive and social/behavioural development (Sammons et al., 2008). Several countries, i.e. Ireland, Greece, Malta, Romania, Finland, the United Kingdom (Scotland) and Norway, emphasise the involvement of parents in the learning process and provide examples of concrete initiatives related to mathematics.

In **Ireland**, the Primary School Curriculum (1999) and the Learning Support Guidelines (2000) ([26]) published by the Department of Education and Skills, as well as initiatives aimed at supporting numeracy in disadvantaged areas (e.g. Maths for Fun teaching strategy) all emphasise the need to build partnerships and empower parents.

Teachers in **Greece** are encouraged to write letters to parents informing them about the content of mathematics classes, the knowledge to be acquired and the targets to be achieved. They may also suggest how parents can develop activities to share with children at home.

The involvement of parents in the learning process in **Romania** is mainly focused on early primary education, and it aims to make parents aware of the role of mathematics in students' cognitive development and to recommend methods that can track the progress and development of students' mathematical abilities.

The Foundation for Educational Services (FES) in **Malta** provides guidance for parents of children at primary school. Parents are encouraged to learn and practice methods that stimulate their children to learn more effectively. They have the opportunity to meet and discuss educational strategies with teachers twice a week. They also join their children in practicing some of the methods. After participating in this process, many parents decide to engage in other non-formal learning opportunities offered by FES and other organisations. An additional activity is the so-called parent-to-parent

([25]) http://www.naturfagsenteret.no/c1515601/prosjekt/vis.html?tid=1519408
([26]) http://www.education.ie/servlet/blobservlet/learning_support_guides.pdf?language=EN

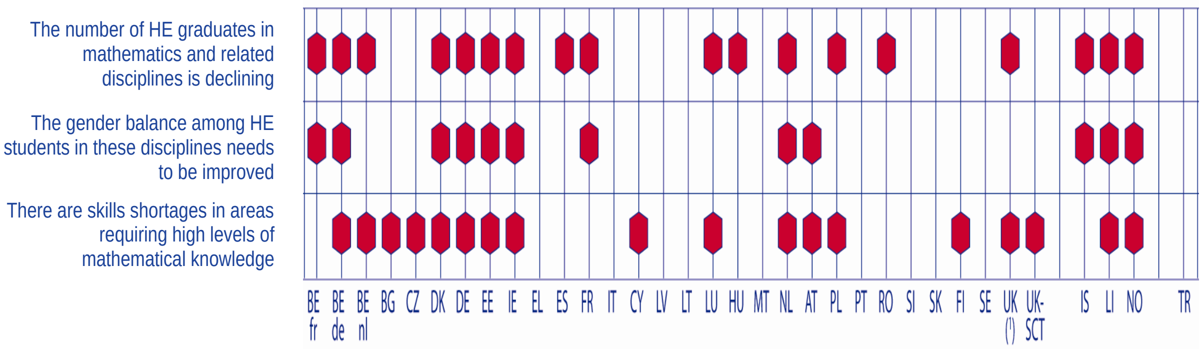
initiative which supports parent empowerment activities. A team of parent leaders is formed and trained to provide courses for other parents under teachers' supervision and guidance ([27]).

The **United Kingdom** (**Scotland**) has passed the Act on Parental Involvement which aims to encourage parents to develop their children's learning at home and in the community. In addition, the Act reflects the shared role and responsibility that schools, parents and carers have in working together to educate children. Across Scotland, parents have been building closer links with schools over the last few years. They are also involved in the life of schools via Parent Councils ([28]). The document 'Learning together: Mathematics' highlights the important role parents play in the development of mathematics and the important role mathematics plays in increasing life chances. The initiative to engage parents in home study includes a workshop to share course content and approaches to learning (HM Inspectorate of Education, 2010). Parents receive a pack of activities containing quizzes, games and sets of questions. Moreover, they can use the school website to download support materials and resources to help their children.

5.4. Policy issues related to skills shortages and the take-up of mathematics in higher education

An important reason for increasing motivation at primary and secondary level, apart from the general improvement of numeracy, is to encourage the choice of mathematics and related subjects at tertiary level. Recent statistical data (see Figure 5.5) shows declining numbers of MST students across Europe. Moreover, several countries indicate a shortage of personnel highly qualified in mathematics and related fields, which can impact on the competitiveness of their economies.

Figure 5.4: Policy concerns related to skills shortages and the take-up of mathematics and related disciplines in higher education, 2010/11



Source: Eurydice.

UK ([1]) = UK-ENG/WLS/NIR

Education authorities in eighteen countries or regions expressed concern about skills shortages in areas requiring high levels of mathematical knowledge. The same number, although a different set of countries or regions, highlighted declining numbers of higher education graduates in mathematics and related disciplines as an important concern. Another issue raised is the need to improve the gender balance among higher education students in MST subjects. However, ten countries did not indicate that any of these issues was a pressing concern and hence do not identify them as a potential problem area in the near future. Iceland and Liechtenstein confirm that these issues are areas of policy concern; however, no measures to address the situation have been defined or planned so far.

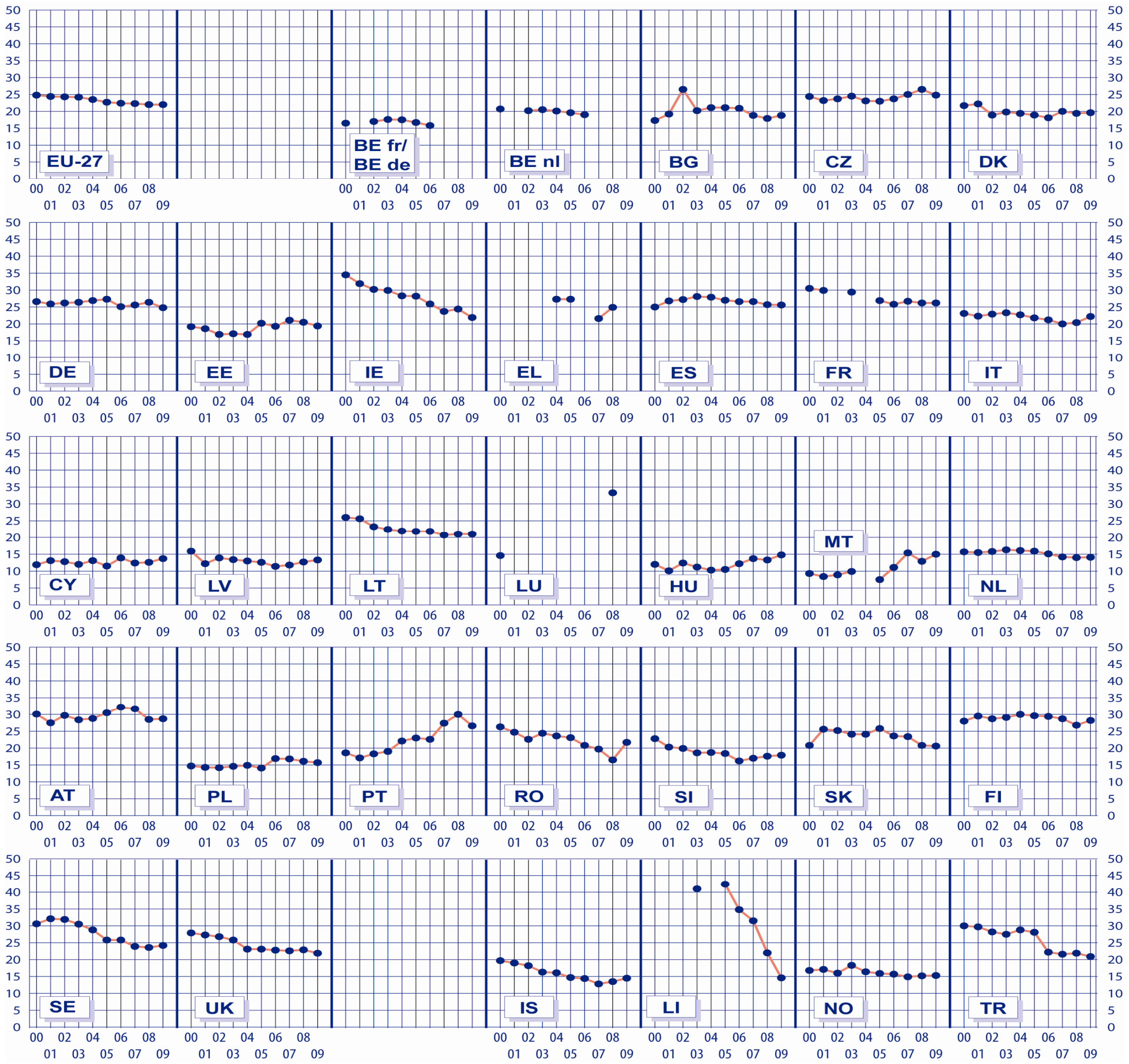
([27]) http://www.education.gov.mt/edu/other_org/fes.htm#The%20Parents-in-Education%20Programme

([28]) http://www.ltscotland.org.uk/parentzone/getinvolved/parentalinvolvementact/index.asp

MST graduate numbers

With growth of over 37 % in the number of MST graduates between 2000-2008, the European Union has already progressed at more than twice the rate foreseen by the EU benchmark (aiming for an increase of at least 15 % by 2010) in this field (European Commission, 2011). However, this growth can be seen to be largely due to the general increase in the number of tertiary students in the EU over the last decade. When analysing the share of MST graduates compared to all university graduates a different picture emerges. In fact, the percentage of MST graduates compared to the total number of graduates in the European Union is decreasing, which is raising concerns not only among education authorities but also among businesses. National authorities are trying to counteract this as they identify the need to maintain a high number of MST graduates as a crucial factor for their competitiveness in the global economy.

Figure 5.5: Percentage of MST graduates (ISCED 5-6), 2000-2009



Source: Eurostat.

Country specific note

Liechtenstein: The figure illustrates only the number of graduates studying in Liechtenstein. There is only a limited offer of study programmes in Liechtenstein. Therefore almost 90 % of students study abroad.

In the European Union, on average, the percentage of graduates in MST fields is steadily declining, from 24.8 % in 2000 to 22 % in 2009 (see Figure 5.5). Compared to 2000, the majority of countries experience a decrease in the number of MST students. Countries with large decreases include Ireland, Lithuania, Romania, Sweden, the United Kingdom, Iceland, and Turkey. A clear tendency for increasing numbers can only be observed in Portugal. The lowest percentages of MST graduates in 2009 (14 % and lower) can be found in Cyprus, Latvia and the Netherlands; while the highest rates of MST graduates (around 28 %) can be found in Austria and Finland.

Some European countries report monitoring the number of MST students and voice concerns over the decrease in graduate rates:

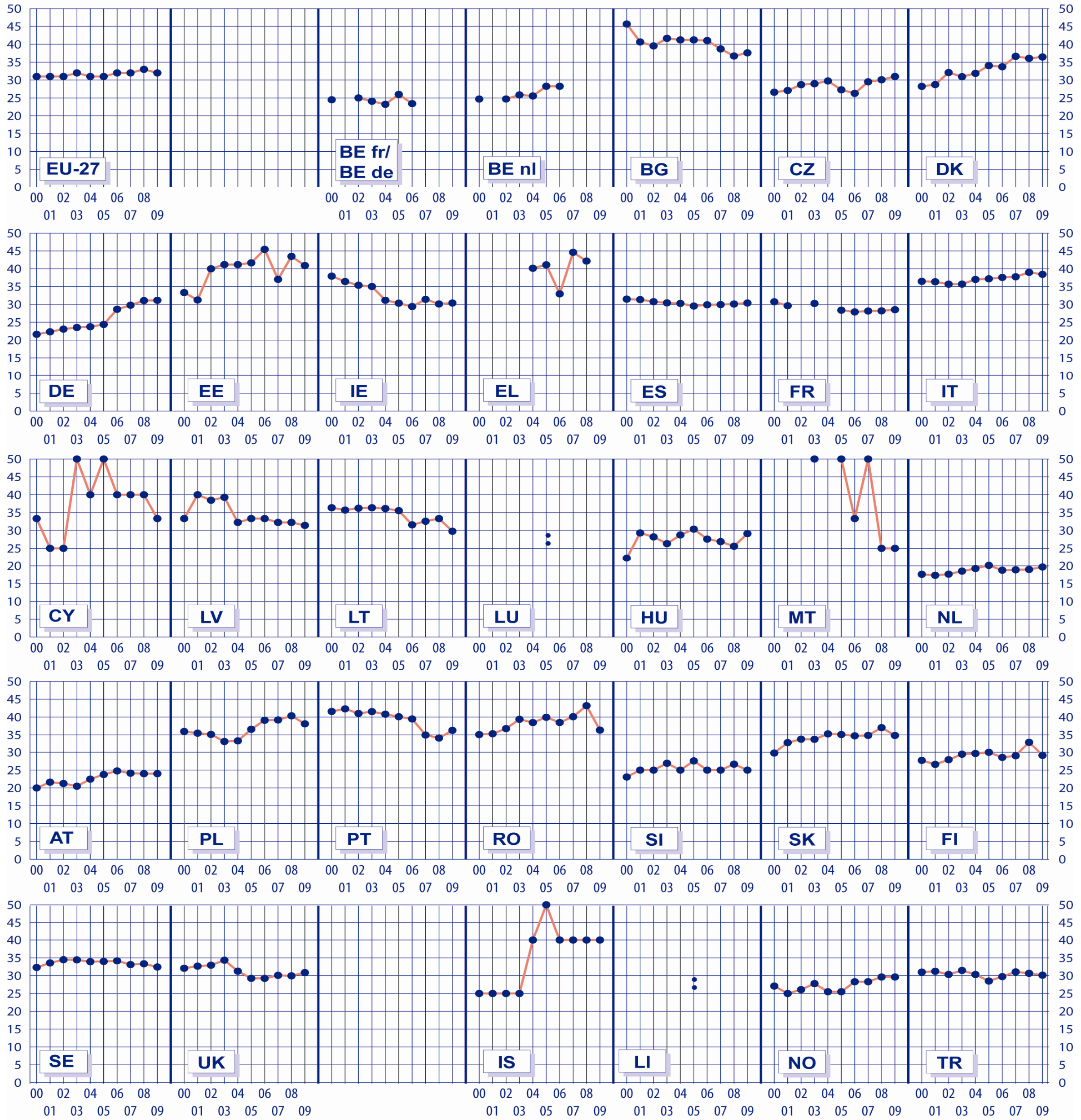
> The **Danish** University and Property Agency (DUPA) provides specific national data on natural sciences, including mathematics, and demonstrates that the situation in this particular field is improving despite a decreasing rate of MST students overall. The completion rates of Bachelor's degrees in the natural sciences increased from 60 % in 2001 to 67 % in 2008. However, the total average number of graduates at Bachelor level in 2008 was higher and reached 74 %. The completion rate at Master's level in the natural sciences was constant at 85 % in the same year. The 2010 intake shows a substantial overall increase in natural sciences of 18 %. It was the highest increase in any field of study. This fact thus reduced the level of concern amongst policy-makers.
>
> In contrast, only 5.2 % of the total number of university students in **Latvia** study natural sciences and mathematics. A lack of MST graduates is also seen in **Poland**. The Ministry of Science and Higher Education provides special funds for mathematics faculties and scholarships for the best students in order to boost the number of graduates in MST. In **Belgium (Flemish Community)**, the Action Plan Science Communication was adopted which defines objectives aimed at increasing the numbers of higher education graduates in mathematics and related disciplines by improving perceptions and attitudes towards these subjects. **France** reports that only 42 % of pupils who choose science as a subject for their school-leaving examination continue in science-related disciplines at higher education level. This represents a decrease of 15 points in ten years. The only mathematics-related field retaining stable numbers in universities is computer science.
>
> Although the **United Kingdom** as a whole has experienced a decline in the numbers of students studying MST subjects at degree level, in **Scotland** higher education institutions report that the number of new students enrolling for mathematics-based degrees is healthy and that they are as able and motivated as previous generations. Some concerns were nonetheless raised regarding perseverance and determination of present generation of students.

Gender balance

Concerns about the gender balance among higher education students in mathematics and related disciplines have been reported by twelve European countries or regions. This is fewer than those highlighting fears about skills shortages or the overall number of higher education graduates in those disciplines. However, countries expressing concern about one of these two issues also indicate a gender imbalance.

According to Eurostat data (see Figure 5.6), the percentage of females as a proportion of all MST graduates in the EU-27 increased over the past years only slightly, from 30.8 % in 2000 to 32.1 % in 2009. A proportion of female MST graduates or around 40 % (in 2009) can only be found in Estonia and Iceland. The Netherlands, on the other hand, has the lowest share of female graduates in MST (19.7 %), followed by Austria (24 %). The largest increase in the percentage of female MST graduates over the last years can be noted in Denmark, Germany and Iceland.

Figure 5.6: Evolution of the percentage of female graduates in the field of mathematics and statistics (ISCED 5-6), 2000-2009



Source: Eurostat.

A few countries are trying to address the imbalance of MST students in general and female students in particular:

Denmark has adopted a strategy for attracting more women into studying mathematics, and a rise from 28.24 % in 2000 to 36 % of female MST graduates in 2007 was noted. **Norway** states a goal of a 15 % increase in MST students in its 'Strategy for Strengthening MST 2010-2014'.

The **Netherlands** with the lowest share of female graduates in MST in the European Union launched a media campaign to encourage girls to choose MST subjects in their school career. The technical universities have initiated projects to attract both men and women into technical studies as the percentage of MST graduates is among the lowest in Europe reaching only 14 % in 2008.

France encourages girls to take up MST studies via national campaigns, but the proportion of female students remains at about 35 % of total number of MST students.

Skills shortages

Skills shortages in the field of MST and particularly in mathematics are reported by several countries. These shortages relate to students' difficulties in mathematics as well as, in some countries, a lack of specialisation among teachers. To this end, some countries have developed measures to improve mathematics teaching and learning and therefore improve students' motivation to learn and study the subject at higher levels.

Norway has adopted a number of measures for strengthening students' skills before they enter higher education. The National Centre for Recruitment in MST plays a vital role in the implementation of these measures.

Employers in the **Czech Republic** point to the fact that the quality of students in higher education institutions largely depends on their level of training at secondary school (National Education Fund, 2009). The ongoing curriculum reform should lead to an improvement of MST instruction in schools. Moreover, business organisations in the Czech Republic support the introduction of a state school leaving examination that would help increase the level of mathematical knowledge essential for the technical and science professions. Pupils took this new form of secondary school leaving examination for the first time in the 2010/11 school year.

Similarly, in **Ireland**, in accordance with the targets outlined in the draft National Plan to Improve Literacy and Numeracy in Schools, 'Better Literacy and Numeracy for Children and Young People' (2010 ([29])), the Department of Education and Skills plans to raise the achievement level of students in the mathematics Ordinary Level examination at the end of the junior cycle, and increase the take-up of Higher Level mathematics in the state examination at the end of the junior cycle (to 60 % by 2020) and at the end of the senior cycle (to 30 % by 2020).

In **Estonia**, several measures have been introduced to counteract the current state of affairs regarding mathematics skills. The University of Tallinn provides special courses for teachers' professional development enabling them to specialise in teaching mathematics at primary schools. The new teaching methods should prevent the fall in results in mathematics exams which has led to lower numbers of students choosing mathematics for further studies. As students' mathematical knowledge is in many cases not sufficient for university studies, schools organise special courses for students to help them reach the required level. Furthermore, in order to counteract the problem of a lack of young teachers willing to teach in non-urban regions, the government provides additional financial incentives.

As in Estonia, additional classes for students with insufficient mathematics skills are also organised by universities in **Poland**. Raising the level of mathematical knowledge in **Bulgaria** is subject to discussion, and a strategic plan to promote mathematics throughout the education system in **Belgium** (**German-speaking Community**) is to be implemented.

([29]) http://www.education.ie/servlet/blobservlet/pr_literacy_numeracy_national_plan_2010.pdf

Summary

Mathematics is one of the basic competences and fundamental for lifelong learning. Motivating students to learn mathematics is crucial for raising their achievement levels in school as well as improving their opportunities to pursue higher academic studies and possibly a career in a mathematics-related field.

Students who have positive attitudes and self-confidence in mathematics usually achieve better results. TIMSS data confirms that in the participating EU countries, especially at eighth grade, those students who had positive attitudes scored higher than those who had negative attitudes. Moreover, TIMSS results showed that achievement is higher among students who perceived mathematics as advantageous for their education and career. It is worth considering how this might be affected by the extent to which they perceive mathematics teaching at school to be linked to their daily life.

The international and national surveys and reports presented in this chapter point to the fact that motivation in mathematics declines over the years spent in the education system and thus measures to counteract this situation need to be developed. Some countries have adopted strategies and initiatives that aim to engage students and increase their interest and active participation in mathematics learning from an early age. They include innovative teaching methods, school partnerships with universities or businesses, and extra-curricular activities targeting talented students in particular. Few countries start these activities in pre-primary education.

Gender specific issues need to be tackled as girls show more anxiety and less confidence in their abilities than boys. Both PISA and TIMSS data reveal that although the gender gap in achievement is not significant, the difference in self-confidence and self-efficacy remain large. Female graduates are underrepresented in MST-related studies and this has not changed considerably over recent years.

Many countries address the issue of motivation in the broader context of MST rather than solely in mathematics. This becomes particularly evident when examining the projects and partnerships promoted in many countries. In addition, policy initiatives at the European level usually address MST as a whole. This approach can be useful; however, equal attention must also be paid to specific subject areas, such as mathematics, to develop targeted strategies for enhancing learner motivation.

A number of national activities for improving students' perception of mathematics learning are focused on talented students rather than on raising motivation among students in general. Students with difficulties in the subject may benefit greatly from additional support and therefore initiatives for improving motivation for learning mathematics could be very effectively targeted at this group.

Those who are motivated and perform well in mathematics at primary and secondary levels are more likely to consider taking up higher education studies as well as career opportunities in MST subjects. Consequently, national authorities in most countries have made raising the number of MST students an important policy objective and are taking measures to improve the situation. The common goal is to support a sufficient number of high quality graduates who will ultimately help Europe to maintain its position in the global economy.

CHAPTER 6: EDUCATION AND PROFESSIONAL DEVELOPMENT OF MATHEMATICS TEACHERS

Introduction

Effective mathematics teaching depends to a large extent on the expertise of teachers; consequently their knowledge of the subject – of mathematical principles and processes – and their professional training are crucial. Good teaching is reliant not only on teachers' mathematical subject knowledge and skills, but also on their understanding of how to teach their subject and of how students learn – both of which are essential if teachers are to reflect on and respond to the needs of their students. Mathematics teachers therefore need to develop and apply sound knowledge and understanding of pedagogy as well as mathematics as a subject.

There is widespread agreement about the link between the quality of teaching and teacher education on the one hand and student attainment on the other, including in mathematics (see for example: Aaronson et al., 2007; Bressoux, 1996; Darling Hammond et al., 2005; Greenwald et al., 1996; Kane et al., 2008; Menter et. al., 2010; Slater et al., 2009; Rivkin et al., 2005). The European Union has also long recognised this relationship and considers the support and development of teachers as an important feature of Europe's education systems ([1]) (European Commission, 2007).

This chapter aims to highlight some of the key aspects of mathematics teacher education and professional development that enable teachers to provide students with the high-quality learning opportunities necessary for high achievement. To this end, it analyses central regulations, recommendations and guidelines related to the structure and content of programmes for mathematics teacher education and professional development. It starts with a profile of the mathematics teaching profession, followed by an analysis of existing policies and practices in European countries regarding initial teacher education (ITE) and continuing professional development (CPD). These are presented against the background of the academic research literature in the field as well as data from the TIMSS and PISA international surveys. In addition, the last section presents some results of a pilot field survey conducted by EACEA/Eurydice on existing practices in the initial education of science and mathematics teachers in several European education systems.

6.1. Demographic challenges for the mathematics teaching profession in Europe

Despite the important role of teachers in the teaching and learning process, the profession as such is currently facing a number of challenges. In a survey by the OECD (2005) on attracting, developing and retaining effective teachers, many countries reported, amongst other issues, concerns about the ageing of the teaching profession, the supply of good quality teachers, the unequal gender distribution, and the weak links between teacher education, teachers' professional development and school needs.

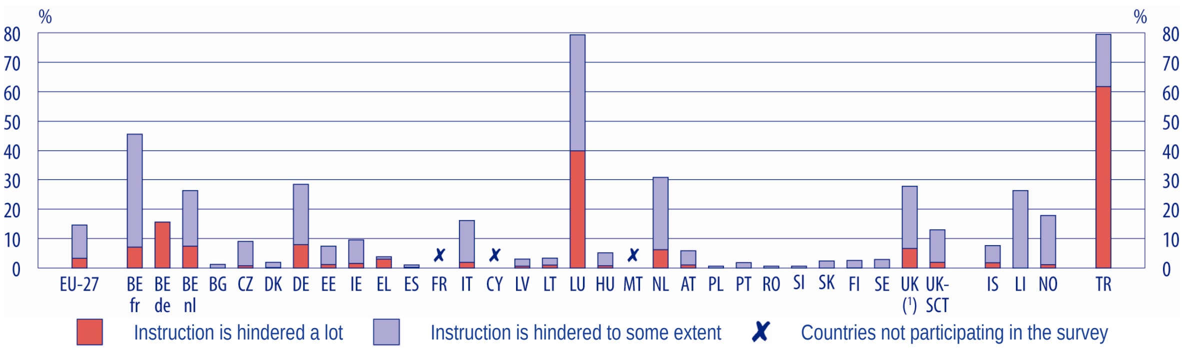
An analysis of the profile of the mathematics teaching body in Europe reveals a similar picture. Several countries report concerns about the supply of mathematics teachers, in particular at lower secondary level:

([1]) Council conclusions of 26 November 2009 on the professional development of teachers and school leaders. OJ C 302, 12.12.2009, pp. 6-9.
Conclusions of the Council and of the Representatives of the Governments of the Member States, meeting within the Council of 21 November 2008 on preparing young people for the 21st century: an agenda for European cooperation on schools, OJ C 319, 13.12.2008, pp. 20-22.
Conclusions of the Council and of the Representatives of the Governments of the Member States, meeting within the Council of 15 November 2007 on improving the quality of teacher education, OJ C 300, 12.12.2007, pp. 6-9.

> **Austria** and **Norway** report a general lack of teachers, including mathematics teachers. In **Belgium (Flemish Community)**, **Germany** and **Ireland**, there are concerns about the lack of qualified mathematics teachers. In the **Netherlands**, there is a shortage of teachers for arithmetic, and more generally there is a need for more know-how in mathematics instruction at lower secondary level.

Data from the latest PISA survey (see Figure 6.1) confirms that some European countries face shortages of qualified mathematics teachers. On average 15 % of all 15-year-old students are taught in schools where the school head reports that teaching is, at least to some extent, hindered by a lack of qualified mathematics teachers. Luxembourg and Turkey are the most affected by this problem, with approximately 80 % of 15-year-olds attending schools where the heads reported this was a problem. These countries were followed by Belgium (French and Flemish Communities), Germany, the Netherlands, the United Kingdom (England, Wales and Northern Ireland) and Liechtenstein, where between 50 and 20 % of students have school heads who reported a lack of qualified mathematics teachers. Around half of European countries do not face any major problems in this area.

Figure 6.1: Percentage of 15-year old students whose principals reported that their school's capacity to provide instruction was hindered by a lack of qualified mathematics teachers, 2009



	EU-27	BE fr	BE de	BE nl	BG	CZ	DK	DE	EE	IE	EL	ES	IT	LV	LT	LU
Instruction is hindered a lot	3.3	7.1	15.6	7.4	0.0	0.8	0.2	7.9	1.2	1.6	3.0	0.2	1.9	0.6	1.0	40.0
Instruction is hindered to some extent	11.3	38.5	0.0	18.9	1.2	8.3	1.8	20.5	6.2	8.0	0.8	0.9	14.2	2.4	2.4	39.4

	HU	NL	AT	PL	PT	RO	SI	SK	FI	SE	UK (1)	UK-SCT	IS	LI	NO	TR
Instruction is hindered a lot	0.8	6.2	1.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	6.6	1.9	1.8	0.0	1.1	61.8
Instruction is hindered to some extent	4.4	24.5	4.8	0.6	1.8	0.7	0.6	2.4	2.5	2.9	21.1	11.1	5.8	26.3	16.7	17.7

Source: OECD, PISA 2009 database.

UK(1): UK-ENG/WLS/NIR

Explanatory note

The figure summarises principals' responses to the option 'lack of qualified mathematics teachers' as part of the question 'Is your school's capacity to provide instruction hindered by any of the following issues?' It shows two out of the four available answer categories ('not at all', 'very little', 'to some extent' and 'a lot').

Country specific note

Austria: The trends are not strictly comparable, as some Austrian schools have boycotted PISA 2009 (see OECD 2010b). However, Austrian results are included in the EU-27 average.

The supply of qualified mathematics teachers is also related to the question of educational background. Results from the TIMSS 2007 international mathematics study provide further information about the educational level of mathematics teachers. In participating EU countries, on average (2) 75 % of fourth grade students and 93 % of eighth grade students had teachers with a university degree. Fifteen per cent of fourth graders and 30 % of eighth graders had teachers who had

(2) Here and elsewhere the Eurydice-calculated EU average refers only to the EU-27 countries which participated in the survey. It is a weighted average where the contribution of a country is proportional to its size.

completed a postgraduate university degree (for example, Master's or Doctorate). However, at the fourth grade, there were some disparities between countries. For example, in Italy, the majority of students had teachers who had completed only secondary education, while in Austria the majority had teachers who had completed a form of tertiary education but not university. At the eighth grade, only Slovenia had approximately half of students who had teachers who had completed post-secondary education but not university (Mullis et al. 2008, pp. 248-49).

Other concerns reported by European countries relate to the age structure of the mathematics branch of the teaching profession:

> While in **Estonia**, insufficient numbers of younger teachers have been reported, the situation in **Finland** is that the average age of mathematics teachers is higher than that of all other subject teachers. In **Romania** and the **United Kingdom** (**Scotland**), analyses have shown that the ageing of mathematics teaching staff will pose problems in the near future. Many teachers will be eligible to retire in the next few years and this poses a threat to the supply of qualified mathematics staff.

In this context, the TIMSS data confirm that in all the participating EU countries most students at fourth and eighth grades (37 % and 45 %, respectively) were taught mathematics by teachers who were aged 50 or older. Amongst those, around 5 % of students had teachers who were aged 60 or older. More than half of students had teachers aged 50 or older in Germany at the fourth grade and similarly in Bulgaria, Italy and Romania at the eighth grade. Relatively few students in the fourth and eighth grades, around 10-15 % on average in the EU, were taught by teachers who were 29 years or under. More fourth grade students had younger teachers in the Netherlands, the United Kingdom (England and Scotland). Approximately 50 % of eighth grade students were taught by teachers who were 29 years or younger in Cyprus and Turkey (Mullis et al. 2008, pp. 244-45).

The TIMSS data on the age patterns of mathematics teachers reveals largely the same trends as can be found in the Eurostat data (reference year 2007) covering the entire teaching workforce in primary and secondary education. It shows that, in the majority of countries, primary and secondary school teachers in the 40 to 50 age group account for the highest proportion of teachers across European countries.

With respect to gender, amongst the European countries, only Estonia reported that mostly female teachers were teaching mathematics. However, according to the TIMSS results, the great majority of fourth grade students were taught mathematics by females (EU average of 84 %). Only Denmark equalled the EU average; while in Italy, Latvia, Lithuania, Hungary and Slovenia more than 95 % of students had female teachers (Mullis et al. 2008, p. 244). This was less so at the eighth grade (EU average of 68 %) where in half of the participating EU countries the proportion of students with female teachers was between 40 and 60 % (ibid., p. 245).

Again, Eurostat data from 2007 on the proportion of female teachers as a percentage of all teachers reflects similar trends to those mentioned above. On average in Europe, 83 % of all teachers at primary level are females. Denmark is amongst the countries with the lowest percentage of female teachers (68 %). At secondary level, the European average is lower than at primary level with 66 % of female teachers, but it remains relatively high in several countries (above 80 %) including Bulgaria, Estonia, Latvia and Lithuania.

Overall, the issues raised above suggest that a number of measures should be taken across Europe to recruit and retain sufficient numbers of qualified men and women – particularly in the younger age groups – to the mathematics branch of the teaching profession. In addition, professional development opportunities can play a key role in equipping all teachers with the necessary skills to adapt their teaching to changes and developments in mathematics education. Across Europe, specific reforms targeting mathematics teachers can be found only in two countries:

> In **Ireland**, teachers teaching mathematics without specialist qualifications are being encouraged to take a post-graduate diploma in mathematics designed in partnership between the Department of Education and Skills and one of the Irish universities. Moreover, the Department of Education and Skills' Draft National Plan to Improve Literacy and Numeracy in Schools ([3]) includes proposals to set new, higher standards in entry requirements for ITE programmes; to re-configure the content and duration of ITE programmes for primary and post-primary teachers; to provide continued support to newly qualified teachers of numeracy and make participation in the national teacher induction programme mandatory by 2012; and to focus provision of CPD on numeracy and the use of assessment.
>
> As a result of the Williams Review (2008), conducted in the **United Kingdom** (**England**), which proposed that a primary mathematics specialist be trained so that there is one in every primary school (or in a cluster of very small schools), the Government designed and supported the roll out of the 'Mathematics Specialist Teacher' programme. The ambition at the time of the launch was that every primary school would have access to a Mathematics Specialist Teacher by 2019.
>
> Moreover, the 'Chartered Mathematics Teacher' scheme (IMA, 2009) has been introduced to raise the status and the professionalism of mathematics teachers. It aims to give added recognition to the profession in a similar way to some other professions, e.g. chartered engineers and chartered surveyors. The status is also available for primary teachers. It puts emphasis on continuing professional development, with a requirement of at least 30 hours per year. Teachers will need to belong to at least one of several mathematics teaching associations and demonstrate subject knowledge as well as knowledge and experience in pedagogy.

In some other European countries, general reforms of university education which also affect mathematics teachers' initial training system are ongoing.

> For example, in **Spain**, the main new developments with regard to initial training for primary education teachers are that prospective teachers must complete a four-year Bachelor degree (240 ECTS), as compared to the previous three-year degrees. Teachers in secondary education and vocational training institutions must, after obtaining the Bachelor certificate, complete an official one-year Master's course (60 ECTS). The previous requirement was for pedagogical and didactic training in the form of 150- to 300-hour courses provided by universities.
>
> In **Iceland**, a new law was passed according to which the requirements for initial teacher training will change in 2011. It will then be necessary to complete a Master's degree consisting of 300 ECTS or equivalent education and training to become a qualified pre-primary, compulsory or upper secondary teacher.

In all countries, teacher education and training as well as working conditions are a subject of general ongoing discussion, and this may also involve mathematics teachers. However, more targeted measures to address the particular challenges faced in the field of mathematics, as outlined above, may be necessary in order to make significant improvements to mathematics teaching in schools in Europe.

6.2. Getting the balance right in the content of initial teacher education

The literature on mathematics teacher education highlights the importance of balancing teachers' subject knowledge in mathematics with their pedagogical knowledge. In its position statement concerning highly qualified mathematics teachers, the US National Council of Teachers of Mathematics (NCTM, 2005) states that they must have 'an extensive knowledge of mathematics, including the specialized content knowledge specific to the work of teaching, as well as a knowledge of the mathematics curriculum and how students learn'. In other words, in addition to a 'profound understanding of fundamental mathematics' (Ma 1999, p. 19), teachers must also have what has been

([3]) http://www.education.ie/servlet/blobservlet/pr_literacy_numeracy_national_plan_2010.pdf?language=EN coined by Shulman (1986) as 'pedagogical content knowledge', i.e. the practical understanding of how to apply their knowledge and adapt it to their teaching, as well as 'curriculum knowledge', which refers to the knowledge of the content, materials and resources used for teaching, how they are arranged and ways to use them.

Many subsequent researchers have continued to develop the notion of teacher knowledge by highlighting additional elements. These include 'context knowledge', which allows teachers to adapt their knowledge to specific settings and students (Grossman, 1990), and 'knowledge of students' cognitions', which allows teachers to understand how students think and learn (see e.g. Fennema & Franke, 1992; Cochran et al., 1993).

In the following sections, the two main aspects of knowledge for mathematics teachers will be examined in more detail: firstly, teachers' knowledge of mathematics as a subject, with particular focus on the differences in the initial education of generalist and specialist teachers; and subsequently, their pedagogical knowledge as it relates to mathematics. Central-level guidance for ITE programmes will form the basis of this analysis.

6.2.1. Knowledge of mathematics as a subject

The development of teachers' subject knowledge (knowledge of mathematical principles and processes) deserves some reflection. Across European countries, mathematics is usually taught by generalist teachers at primary level. The exceptions are Poland where mathematics is taught by specialist teachers at the second stage of primary education (grades 4-6), and Denmark where teachers at primary level are specialised in up to four 'main subjects'. At lower secondary level, mathematics is taught by specialist mathematics teachers and/or semi-specialist teachers (qualified to teach two or three other subjects in addition to mathematics).

The current situation has led to serious concerns in some European countries, as reported for example by the United Kingdom, about the level of specialist knowledge required of generalist teachers teaching mathematics at primary level. In the majority of countries, where centrally defined regulations or recommendations concerning ITE identify the minimum proportion of the course load that should be dedicated to developing prospective teachers' knowledge of mathematics, the percentages are indeed much higher for specialist mathematics teachers than for generalist teachers (see Figure 6.2). In all other countries, general guidelines for the structure of courses may be provided at central level; however, it is largely left to higher education institutions to determine the proportion of time spent on mathematical subject knowledge and mathematics teaching skills within their programmes.

The differences between the proportion of mathematical subject content for specialist teachers and that for generalist teachers are significant. In Spain, for example, the proportion is 40 % for specialist compared with 7.5 % for generalist teachers; in Lithuania, the percentage ratio is 56:2-3; and in Turkey, 50:4. In Malta, there are no minimum recommendations in ITE for generalist teachers regarding mathematical subject knowledge but there are for teaching mathematical skills, which again is lower than that for specialist teachers.

Figure 6.2: Centrally defined regulations/guidance on the minimum proportion (as a percentage) of course load to be devoted to mathematical subject knowledge and mathematics teaching skills within ITE programmes, 2010/11

Generalist teachers	BE fr	BE de	BE nl	BG	CZ	DK	DE	EE	IE	EL	ES	FR	IT	CY	LV	LT	LU
Maths subject knowledge	:	○	○	○	○	○	○	:	○	○	7.5	2	5	:	○	2-3	:
Maths teaching skills	:	○	○	○	○	○	○	:	○	○		2	3	:	○	2-3	:
	HU	MT	NL	AT	PL	PT	RO	SI	SK	FI	SE	UK (1)	UK-SCT	IS	LI	NO	TR
Maths subject knowledge	○	5	○	2	○	:	○	○	:	○	○	○	4	○	⊗	:	4
Maths teaching skills	○		○	6	○	:	○	○	:	○	○	○	4	○	⊗	:	5
Specialist teachers	BE fr	BE de	BE nl	BG	CZ	DK	DE	EE	IE	EL	ES	FR	IT	CY	LV	LT	LU
Maths subject knowledge	:	○	○	○	○	○	○	:	○	○	40	5	10	14	○	56	:
Maths teaching skills	:	○	○	○	○	○	○	:	○	○		5	10	7	○	25	:
	HU	MT	NL	AT	PL	PT	RO	SI	SK	FI	SE	UK (1)	UK-SCT	IS	LI	NO	TR
Maths subject knowledge	○	33	○	15	90	:	○	○	:	○	○	○	10	○	⊗	:	50
Maths teaching skills	○	23	○	10	10	:	○	○	:	○	○	○	10	○	⊗	:	30

○ No central regulation/recommendation/guidelines ⊗ No initial teacher education

Source: Eurydice.

UK (1) = UK-ENG/WLS/NIR

Explanatory note

The figure presents the minimum proportion (as a percentage) of the course load that should be dedicated to mathematical subject knowledge and mathematical teaching skills, respectively, in ITE programmes, as defined by central level regulations, recommendations or guidelines.

Country specific notes

Spain: There is no distinction in the regulations between mathematical subject knowledge and mathematical teaching skills. The data for generalist teachers refers to provisions at several universities, whereas central regulations only set down the overall proportion of the teacher training course load that must be distributed between the six content areas of primary education (including mathematics). The data for specialist teachers refers only to the Master's degree.
Italy: The data refers to semi-specialist teachers who are responsible for teaching mathematics at lower secondary level.
Austria: The data for specialist teachers teaching at ISCED 2 refer to teachers at the *Hauptschule* not the *Allgemeinbildende höhere Schule* (AHS).
Liechtenstein: No initial teacher education institution.

Data from the TIMSS 2007 international mathematics study confirm the trends identified above. According to the results, teachers of fourth grade students in a number of countries reported little specific training or specialised education in mathematics. Eighty per cent or more fourth grade students in Austria, Hungary, Lithuania, and Slovakia had teachers who had qualified to teach in primary education without any specialist training in mathematics. At the other end of the scale, ca. 70 % of fourth grade students had teachers who had completed initial education for teaching at primary level with a major or specialisation in mathematics in Germany and Latvia (Mullis et al. 2008, p. 250).

At the eighth grade, on average in the EU, most students had teachers who had studied mathematics (59 %) and mathematics education (57 %). Overall, 88 % of eighth grade students had teachers who had studied mathematics or mathematics education (since teachers often reported that their studies were focused on more than one area). Norway is an exception with only 44 % of eighth grade students who had teachers specialised in mathematics or mathematics education; most students had teachers with specialisations in other areas of study (Mullis et al. 2008, p. 251).

6.2.2. Mathematics-related knowledge and skills for teaching

In the context of mathematics pedagogy, Ball and Bass (2000) most notably seek to complement the concept of teaching knowledge by proposing the sub-category of 'mathematical knowledge for teaching'. It refers to the mathematical knowledge that is specific to the profession of teaching – including considering students' mathematical thinking, following topics as they evolve in class, providing new representations or explanations for familiar topics, etc. But it also means planning interactive teaching lessons, evaluating student progress and making assessments, explaining class work to parents, managing homework, addressing equity issues, etc. – all of which must occur against the background of teachers' 'knowledge of mathematical ideas, skills of mathematical reasoning and communication, fluency with examples and terms, and thoughtfulness about the nature of mathematical proficiency' (Ball et al. 2005, p. 17).

Researchers who have examined the knowledge and skills needed to do this work have found that teachers' higher scores on these measures of mathematics knowledge for teaching contribute to gains in student achievement (ibid.; Hill et al., 2005; Hill et al., 2008; Hill, Schilling, & Ball, 2004).

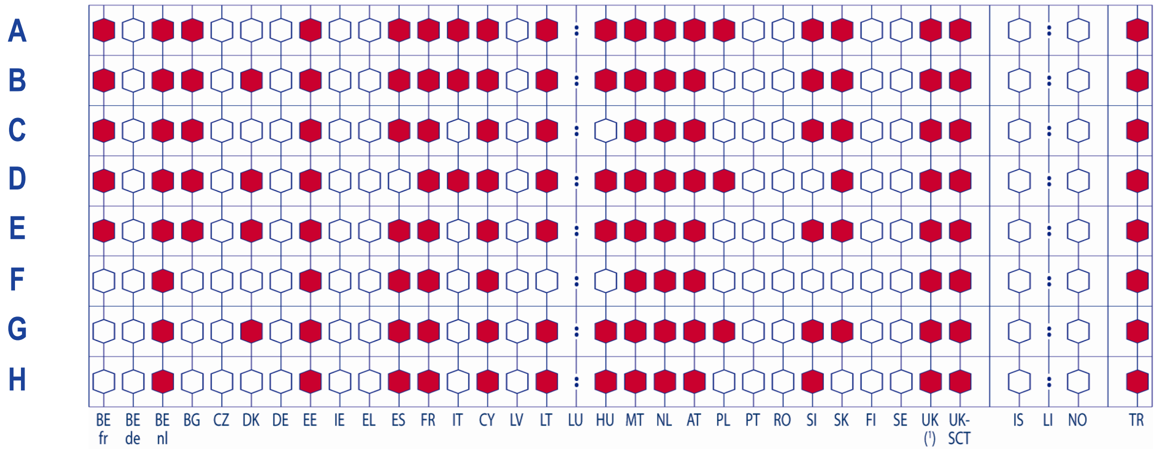
Evidence suggests, therefore, that in order to prepare teachers effectively, ITE programmes need to provide them with a sound understanding of mathematics-related knowledge and skills for teaching. Those European countries that provide central level regulations, recommendations and/or guidelines for ITE programmes already specify a number of areas of knowledge which prospective mathematics teachers should cover in their training (see Figure 6.3). However, the issues that are less frequently addressed at central level include gender sensitive mathematics teaching, conducting and using mathematical research, and assessing students in mathematics. In twelve countries or regions, higher education institutions are completely autonomous in determining the content of their mathematics teacher education programmes.

Most countries with central regulations, recommendations and/or guidelines for ITE programmes require that teachers should know how to teach the mathematics curriculum, how to create a variety of teaching and learning situations as well as how to use a diverse range of teaching materials. They should be able to monitor students' learning and their beliefs and attitudes towards mathematics as well as tackle their learning difficulties. To this end, teachers are also required to know how to involve parents and other players such as educational authorities in students' school lives, as well as how to collaborate with peers for sharing knowledge and experiences acquired in the mathematics teaching process.

An example for this is given by the **Danish** Ministerial order on the basic education programme for primary and lower secondary mathematics school teachers ([4]), which determines that teachers should acquire the competence to justify, plan and execute mathematics teaching as well as to identify, evaluate and develop teaching materials with the aim of uncovering students' learning strategies and attitudes towards mathematics, motivating and inspiring their involvement in mathematical activities and tackling their difficulties in mathematics. Moreover, teachers should develop the skills to communicate and cooperate with colleagues and persons outside the school, i.e. parents, administrative and public authorities, regarding issues related to mathematics teaching.

([4]) *Bekendtgørelse om uddannelsen til professionsbachelor som lærer i folkeskolen* (Regulation on the professional Bachelor programme for teachers in the *folkeskole*). BEK nr 408 af 11/05/2009: https://www.retsinformation.dk/Forms/R0710.aspx?id=124492

Figure 6.3: Centrally defined regulations/guidance on the areas of knowledge and skills for mathematics teaching to be covered in ITE, 2010/11



Central regulations, recommendations and/or guidelines | No central-level guidance/full autonomy of HEI

A	Knowing and being able to teach the official mathematics curriculum	E	Tackling pupils' difficulties in mathematics
B	Creating a rich spectrum of teaching/learning situations and teaching materials	F	Teaching mathematics in a gender sensitive way
C	Developing and using a variety of assessment instruments for formative and summative purposes	G	Collaborating with peers, parents, authorities, etc.
D	Identifying and analysing pupils' learning as well as their beliefs and attitudes towards mathematics	H	Conducting research, alone or with colleagues and using research results in daily teaching practice

Source: Eurydice.

UK (¹) = UK-ENG/WLS/NIR

Explanatory note

The figure shows whether central level regulations, recommendations or guidelines for ITE programmes identify any final competences related to the knowledge and skills needed for mathematics teaching that prospective teachers should develop or whether higher education institutions have full autonomy with regard to the content of ITE programmes.

Country specific notes

Austria: Data refers to primary (ISCED 1) and *Hauptschule* teacher training not that of AHS-teachers teaching at ISCED 2 and 3 where universities have full autonomy.
Liechtenstein: No initial teacher education institution.

Around half of all countries with regulations or recommendations for the initial education of mathematics teachers stipulate that they should know how to select and use a range of assessment tools for formative and summative purposes, and how to conduct research and/or make use of research results in their daily teaching practice.

The regulations for the initial education and training of mathematics teachers at secondary level in **Spain**, for example, stipulate that all prospective teachers must have knowledge of assessment strategies and techniques and have an understanding of assessment as an instrument for regulating and encouraging students' efforts. More generally, they must have the skills to plan, develop and assess the teaching and learning process. For this purpose, teachers are trained to understand and apply basic methodologies and techniques of educational research and evaluation and they learn how to design and develop innovative research and evaluation projects.

In only around one-third of European countries with central level regulations or recommendations for ITE programmes, prospective mathematics teachers are explicitly required to know how to teach the subject in a gender sensitive way.

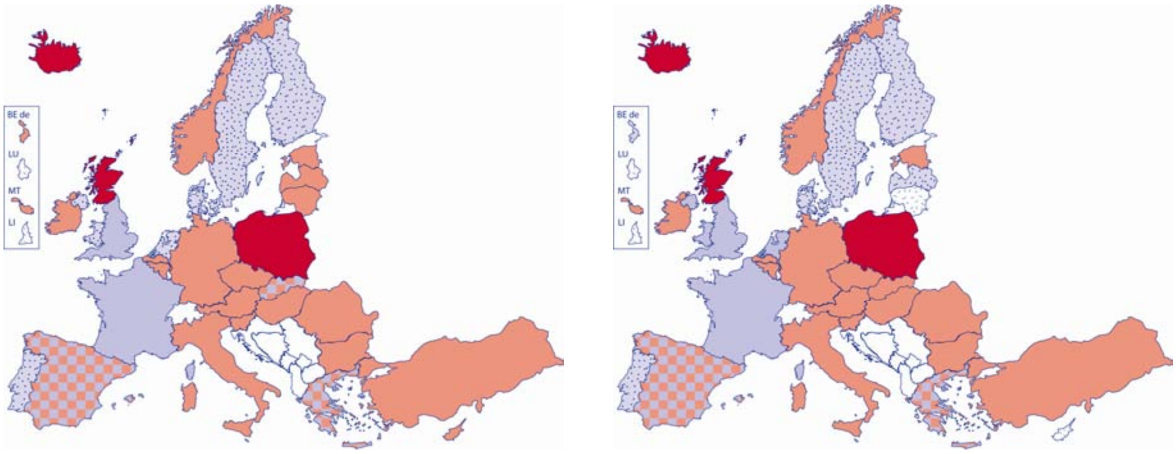
For example, in the **United Kingdom** (**England**, **Wales** and **Northern Ireland**) ([5]) and in **Scotland** ([6]), ITE programmes are guided by general standards for initial teacher education and training that require student teachers at the end of their training to be able to respond to differences between students and to adjust their teaching, expectations and pace of work in such way that it makes appropriate demands on all students. This focus on equality includes gender equality.

6.2.3. Assessment of prospective teachers

In most countries where central level regulations, recommendations and/or guidelines for ITE programmes lay down the areas of knowledge that teachers should develop for mathematics teaching – and also in some countries where higher education institutions have autonomy regarding their ITE programme content – prospective mathematics teachers (specialists and semi-specialists) must be assessed on their *mathematics-related teaching skills*. This occurs mostly in the form of oral and/or written exams, during and at the end of the study programme. However, the content of the exams, their form and evaluation are usually the responsibility of the higher education institutions providing the ITE programmes. In three countries or regions (Poland, the United Kingdom (Scotland) and Iceland), higher education institutions have full autonomy for setting the exams for student teachers.

◆ ◆ ◆ Figure 6.4: Assessment of prospective mathematics teachers, 2010/11

Specialist teachers' mathematics-related teaching skills | **Generalist teachers' mathematics subject knowledge**



Full autonomy of HEI to decide on assessment	Assessments exist, set by HEI	Central exams exist	No central exams exist	Data not available

Source: Eurydice.

Country specific note

United Kingdom (ENG/WLS/NIR): Students must pass a central exam showing a particular qualification in mathematics in order to be accepted onto a teacher training course.

◆ ◆ ◆

A similar situation exists for prospective generalist teachers of mathematics; in around half of European countries, they are assessed on their *mathematics subject knowledge*. This is slightly fewer than the number of countries assessing *mathematics-related teaching skills*. However, prospective generalist teachers are not only assessed during, and at the end of their ITE programme, but frequently also at the beginning in the form of an entrance examination. Again, it is largely the responsibility of higher education institutions to design and organise exams on *mathematics subject*

([5]) Standards and requirements for initial teacher training (ITT): http://www.tda.gov.uk/training-provider/itt/qts-standards-itt-requirements.aspx

([6]) Standards for initial teacher education (ITE): http://www.gtcs.org.uk/web/FILES/the-standards/the-standard-for-initial-teacher-education.pdf *knowledge*; in Poland and Iceland, institutions are fully autonomous with respect to setting such exams for student teachers.

Central exams for *mathematics-related teaching skills* exist only in very few countries:

> In **France**, prospective mathematics teachers need to pass a national competitive exam known as 'CAPES' at the end of their studies. It comprises a written and oral exam as well as an interview by a panel. All parts of the exam are based on the mathematics curriculum for lower and upper secondary education. Moreover, student teachers must demonstrate their mathematical and professional background, their knowledge of mathematics subject content and programmes, and their reflections on the history and purpose of mathematics as well as its relation to other disciplines.
>
> In the **United Kingdom** (**England**), all teacher trainees have to pass skills tests in numeracy (as well as literacy and ICT) before they can begin their induction period. The tests cover the core skills teachers need to fulfil their wider professional role in schools, rather than the subject knowledge required for teaching. The tests must be taken by all new entrants to the teaching profession regardless of the training route followed.
>
> In **Greece**, in addition to mathematics examinations for access to and during their studies at university, prospective teachers are also examined in mathematics when they take the examination of the Supreme Council for Civil Personnel Selection (ASEP). Similarly in **Spain**, apart from the access examinations in mathematics and also during their studies, candidates who want to become primary and secondary education (mathematics) teachers in the public sector must pass a competitive examination organised by each Autonomous Community, in which they must demonstrate their mathematics teaching skills and mathematics subject knowledge. In **Slovakia**, too, at the end of their studies prospective teachers of mathematics must pass the state examination in which both their mathematics teaching skills and content knowledge are assessed. Success in the state examination is the condition for receiving a full teaching qualification.

With respect to *mathematics subject knowledge*, student teachers must pass a central exam in mathematics only in Greece, France and the United Kingdom (England, Wales and Northern Ireland); while in the Netherlands, a central independent assessment body (CITO) has developed a mathematics entrance test for all teachers.

6.3. The importance of ongoing subject-based, collaborative professional development

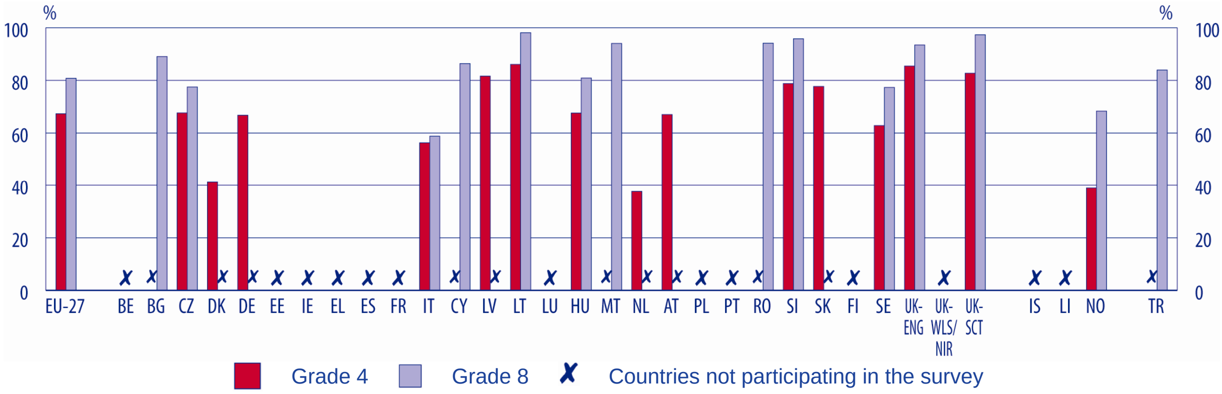
Following their initial education, mathematics teachers must continue to update their knowledge and skills. Opportunities for teachers to engage in continuing professional development (CPD) can have a substantial impact on their work, their achievement, skills and attitudes as well as on their performance and job satisfaction (Villegas-Reimers, 2003). Moreover, the changes to teachers' knowledge and behaviour in the classroom brought about by professional development also have an impact on student learning. An overwhelming amount of research evidence shows that teachers' professional development has a positive effect on student achievement (see, for example, the review by Hattie, 2009).

In the context of mathematics, CPD opportunities are important for generalist teachers who teach mathematics but may not have a background in mathematics or hold specific qualifications in the subject. However, CPD is equally important for experienced and specialised mathematics teachers. Teachers of mathematics must not only deliver the curriculum, but they must be able to adapt their teaching methods to the changing needs of students. They must learn to integrate new materials and technologies, and make use of research results relating to student learning and teaching practices in mathematics (Smith, 2004).

International survey results (see Figure 6.5) show that the take-up of CPD programmes by mathematics teachers at primary and secondary level varies between countries. At the fourth grade, approximately two-thirds of students on average, in the participating EU countries, had teachers who had participated in some type of professional development during the previous two years in the various areas of mathematics specified by TIMSS. Countries where approximately 80 % of students in the fourth school year had teachers who attended at least one form of CPD included Latvia, Lithuania, Slovenia, Slovakia and the United Kingdom. Only about 40 % of students had teachers who attended CPD in Denmark, the Netherlands and Norway.

At the eighth grade, participation in professional development in the specified areas was higher than at the fourth grade. Approximately 81 % of eighth grade students, on average in the participating EU countries, had teachers who had participated in some type of professional development during the previous two years. Participation rates ranged from 59 % in Italy to 98 % in Lithuania.

Figure 6.5: Percentage of students at the fourth and eighth grades whose teachers reported participation in some type of CPD during the previous 2 years, 2007



	EU-27	BG	CZ	DK	DE	IT	CY	LV	LT	HU	MT	NL	AT	RO	SI	SK	SE	UK-ENG	UK-SCT	NO	TR
■	67.3	x	67.6	41.2	66.8	56.3	x	81.6	86.1	67.6	x	37.6	67.0	x	78.8	77.7	62.8	85.5	82.7	39.0	x
■	80.8	89.1	77.5	x	x	58.8	86.4	x	98.1	80.9	94.0	x	x	94.1	95.8	x	77.3	93.5	97.3	68.3	83.9

Source: IEA, TIMSS 2007 database.

Explanatory note

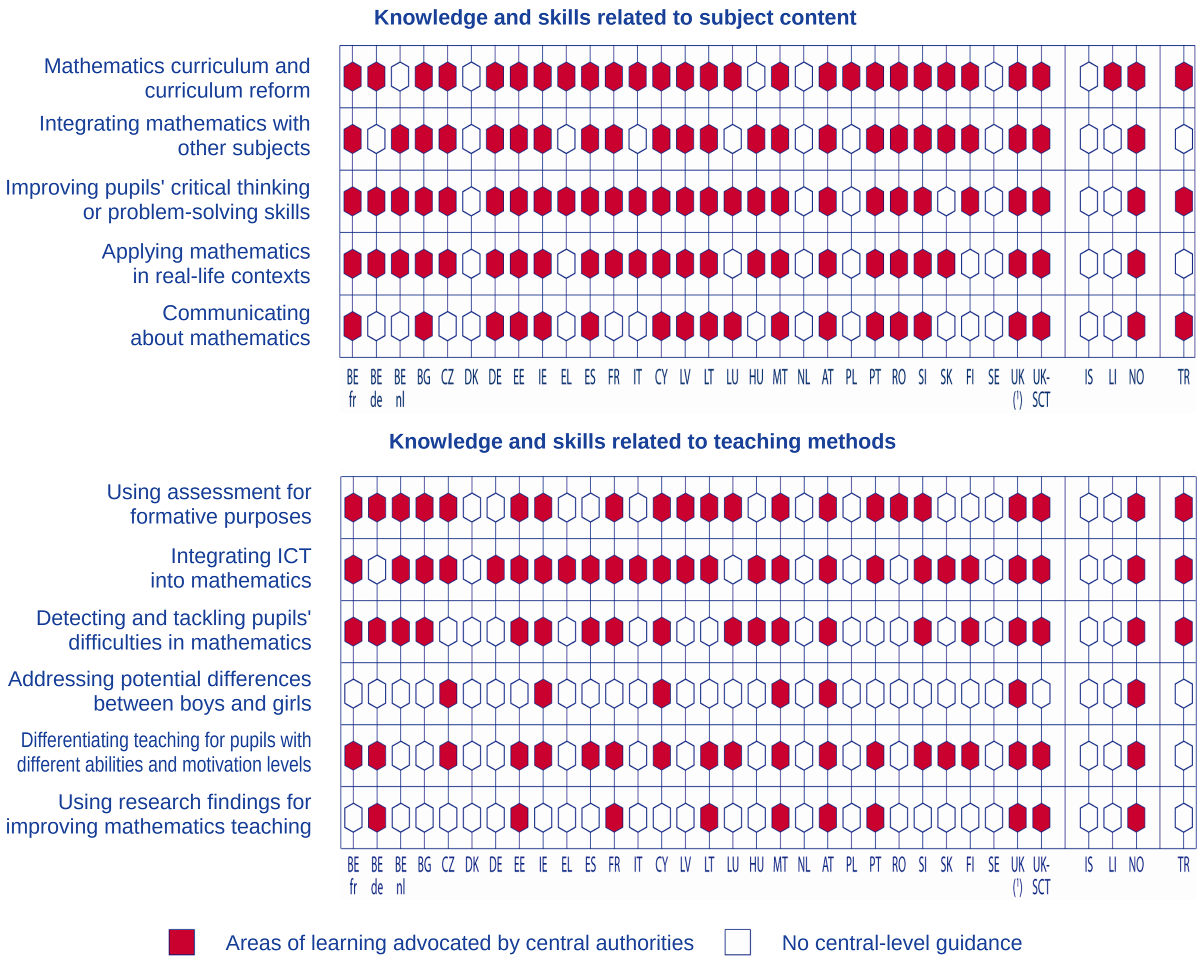
The Figure shows the percentage of fourth and eighth grade students whose mathematics teachers reported participating in at least one form of professional development (CPD) related to mathematics teaching over the previous two years. The CPD areas covered included the mathematics curriculum, subject content, pedagogy/instruction, assessment, integrating information technology into mathematics, and improving students' critical thinking or problem solving skills.

These results may also be seen against the background of data from the OECD's Teaching and Learning International Survey (TALIS). On average, across all European countries participating in the survey, the percentage of lower secondary teachers who undertook professional development in the previous 18 months was around 90 %. The range was relatively small with 75 % in Turkey and 100 % of teachers in Spain who had had some CPD in the 18 months prior to the survey (European Commission, 2010).

In terms of CPD content, research evidence supports the importance of developing specific teaching skills for mathematics, as mentioned above. Timperley et al. (2007), for example, reviewed 72 studies assessing the effects of professional development on student outcomes to identify which aspects of the knowledge and skills delivered during professional development sessions seemed to be most effective. They found that professional development was more effective when it went beyond generic pedagogy by providing teachers with a range of mathematics-based content and teaching methods that were specific and exclusive to mathematics. Elements that appeared to be particularly successful in terms of changing mathematics teaching in ways that led to positive student outcomes included those that engaged teachers in the current research findings that underpinned the particular methods being advocated. Moreover, they shared an emphasis on developing students' conceptual understanding of mathematics and encouraged multiple approaches to mathematical problem-solving. All successful CPD activities developed not only teachers' understanding of their students' mathematical thinking but also their ability to evaluate it. In this way, teachers' decisions could be founded on a deeper knowledge of their learners.

As far as national policies are concerned, European countries cover a wide range of topics pertaining to mathematics teaching through CPD programmes and/or strategies developed at central level (see Figure 6.6). Most countries recommend, in particular, CPD initiatives focusing on enhancing teachers' knowledge related to mathematics subject content. In contrast, fewer countries promote CPD programmes related to mathematics teaching methods, and only a minority of them focus on supporting gender sensitive teaching or involvement in research. Three countries (Denmark, Sweden and Iceland) do not centrally promote the development of teachers' mathematics knowledge through CPD.

Figure 6.6: Knowledge and skills for mathematics teaching to be developed through CPD, as advocated by central authorities, 2010/11



Source: Eurydice.

UK (1) = UK-ENG/WLS/NIR

Explanatory note

The Figure refers to the areas of learning that are either advocated in official documents or through centrally organised CPD courses. However, teachers' participation in such courses is not necessarily compulsory.

Country specific note

Czech Republic: The data refers to CPD courses that have been carried out in the last five years.

The majority of European countries indicate that teachers should develop their knowledge of the school mathematics curriculum and keep up to date with reforms. They should also learn how to integrate ICT into their mathematics teaching and find ways to improve students' critical thinking or problem solving skills in mathematics.

In **Slovenia**, the Ministry of Education and Sport issues an annual catalogue of teacher professional development opportunities. Among the seminars offered is one on 'Mathematics through Research and Problem Solving'. In this seminar, teachers learn about the importance of problem-based knowledge, the transition from closed problems to open-ended problems and the planning and application of problems-based research to different stages of learning.

Similarly in **Spain**, among the courses offered within the programmes of the Autonomous Communities' Teacher Continuing Training Centres, the course on 'Strategies for Mathematics Problem-Solving' offered in Catalonia is aimed at generalist teachers at primary level in particular, and aims to provide them with different methodologies to work with problem-solving in their daily teaching practice.

In terms of actual CPD participation, data from TIMSS 2007 shows that on average in the participating EU countries, at fourth grade, 33 % of students had teachers who had attended professional development courses on improving students' critical thinking or problem-solving skills and 34 % had teachers who had attended CPD courses on teaching the mathematics curriculum. Somewhat fewer fourth grade students had teachers who had undergone professional development in integrating information technology into mathematics (25 %). At eighth grade, the averages were generally higher, with 51 % of students who had teachers who had attended CPD on integrating information technology into mathematics and 42 % on the mathematics curriculum. However, at this educational stage, the proportion of students with teachers who had undergone professional development in improving students' critical thinking or problem solving skills was relatively low with 31 % (Mullis et al. 2008, pp. 252-253).

In a large number of countries, CPD programmes which are centrally organised or advocated provide teachers with understanding of how to integrate mathematics into other subjects or applying it in real-life contexts. The latter is based on the assumption that the learning of mathematics encompasses not just the ability to carry out procedures and develop an understanding of mathematical ideas and how they relate, but also the construction of meaning for the ways in which those mathematical ideas are useful (see for example, Ainley et al., 2006).

In the **Czech Republic**, for example, a course provided by the National Institute for Further Education in 2009 for a limited number of participants had its focus on 'Everyday life in mathematical exercises'. It looked at solving problems in an amusing way, using ideas from real life. It provided secondary level mathematics teachers with a set of relevant activities and exercises to use with their students.

A project promoted in **Estonia** with the title 'We love Maths' had a similar aim of providing specialist teachers at secondary level with information and materials that would help them to identify mathematics problems, which are relevant, interesting and motivating for students.

Around two-thirds of European countries organise or advocate CPD programmes through which teachers develop their knowledge and understanding of assessment for formative or summative purposes.

Malta is one country where a comprehensive CPD programme for primary and secondary level teachers is coordinated by the Directorate for Quality and Standards in Education and provided annually. The professional development programme includes a module on the use of formative assessment at primary level, stressing in particular the importance of constructive feedback to students, the sharing of learning intentions, and self-assessment and questioning techniques.

According to TIMSS 2007 data, teacher participation in CPD opportunities dealing with student assessment is not as widespread at primary level. On average, only 26 % of European students at fourth grade had teachers who had undertaken professional development in mathematics assessment, compared to 43 % at the eighth grade.

Communicating about mathematics, tackling students' difficulties and using differentiated teaching for students of different abilities and levels of motivation are areas of CPD advocated by central authorities in around half of European countries.

> An example of the last area mentioned can be found in the **United Kingdom** (**Scotland**) where CPD courses for all age ranges cover differentiated learning, individualised learning planning and, most crucially, fostering a deep understanding of progression so that teachers can determine where a student is in their mathematical development, decide what the next steps might be, and discuss and plan their implementation.
>
> CPD programmes addressing the issue of tackling students' difficulties in mathematics may be exemplified by the situation in **Belgium**, both in the French and German-speaking Communities. The centrally promoted programmes in this area focus on 'Dyscalculia', which involves training teachers to identify students' particular difficulties in learning or understanding mathematics, to develop strategies to support struggling students, and in cooperation with students, to apply and evaluate both teaching methods and progress.

Despite the growing body of evidence, supporting the use of research to help teachers reflect critically on their practice (see for example a historical review by Breen, 2003), only nine countries or regions advocated CPD programmes which encourage teachers to access and use research findings related to mathematics teaching. Similarly, only a small number of countries explicitly advocate CPD programmes which help teachers to address any possible differences between boys and girls in the teaching and learning of mathematics, as was the case with ITE programmes.

Finally, considering participation in CPD more generally, the training in new approaches, methods and skills needed to implement reform initiatives might reach more teachers if incentives to take part were offered. However, apart from those countries where CPD participation is directly linked with career development and salary increases, only a minority of others offer any real external incentives to encourage teachers to maintain or update their skills through CPD.

> Only three countries or regions – **Belgium** (**Flemish Community**), **Malta** and **Iceland** – report that funds and/or material resources (e.g. laptops) are made available to schools for professional training purposes; in **Finland,** in-service teacher training is offered entirely free of charge to encourage participation.

6.3.1. Collaborative learning

Professional development for mathematics teachers in the areas mentioned above undoubtedly has an important impact on teaching practices inside the classroom. By implementing lessons learned through CPD every mathematics teacher can contribute to improving mathematics teaching in his or her own class. In addition, research increasingly points to the importance of considering the social dimension of learning for teachers, including communication, collaborative learning, sharing of knowledge etc. Without this, it is argued, progress on a larger scale would be difficult to achieve (Krainer, 2003; 2006).

Furthermore, in order to achieve sustainable improvements in mathematics teaching, it appears crucial to support 'communities', i.e. small teams, communities of practice and loosely-coupled networks (Krainer, 2003), where teachers and other relevant players cooperate and collaborate with each other with a view to learning autonomously as well as supporting the learning of others. A particular form of collaborative practice which is frequently described as being effective at improving teaching is 'lesson study', in which groups of teachers meet regularly over long periods of time to work on the design, implementation, testing, and improvement of a specific lesson (Stigler and Hiebert, 1999). This is also applicable to the mathematics classroom (Burghes and Robinson, 2010).

An example of the practical implementation of teachers' collaborative learning is the European project PRIMAS ([7]), supported by the EU's Seventh Framework Programme. It aims to develop and work with networks of teachers and professional development providers across 12 countries in order to support them to promote students' inquiry skills in mathematics and science. The project provides professional development materials to explore effective teaching methods as well as classroom materials for direct use by students; and it ensures that teachers are also supported indirectly through work with a wide range of stakeholders such as parents and policy makers.

At national level, too, the great majority of European countries promote and/or provide support for the development of teachers' networks for exchanging ideas, teaching methods, materials and experiences as well as for fostering cooperation between teachers from different schools or between teachers and researchers ([8]). In around half of these countries, the focus is on providing a variety of formats for meeting and exchanging ideas such as working groups, projects, conferences, seminars, etc.

In **Austria**, for example, under the IMST *(Innovationen Machen Schulen Top)* initiative, mathematics programmes and working groups are organised in each province. They bring together mathematics teachers and academics for initiating, promoting, disseminating, networking and analysing innovations in schools, and offer policy recommendations for a support system for the development of high-quality mathematics teaching at local, regional and national level. Austrian-wide meetings and a newsletter support the exchange of expertise between teachers and academics and foster collaboration.

The **Estonian** Mathematical Society and community of School Mathematics Teachers regularly organises events for mathematic teachers and is extensively used for collecting feedback and suggestions for curriculum development. Moreover, the 'Days of Mathematics Teachers' is an annual event where teachers exchange information about the latest research results, good practices, etc. The presentations given at this occasion are subsequently published in a peer-reviewed journal (*Koolimatemaatika* – School Mathematics).

In **Ireland**, at primary level, a number of Teacher Professional Communities (TPC) have been established through the Teacher Education Network relating, amongst other things, to Maths Recovery. The purpose of a TPC is to enable the collective development of new competences, resources and shared identities as well as motivation to work together for change. At post-primary level, the Teacher Professional Networks collaborate with the Department of Education and Science, the Education Centre Network and the Project Maths Development Team in the design and delivery of ongoing CPD and the organization of conferences and mathematics competitions.

The National Centre for Excellence in the Teaching of Mathematics (NCETM) in the **United Kingdom** (**England**) aims to meet the professional aspirations and needs of all mathematics teachers and realise the potential of learners through a sustainable national infrastructure for mathematics-specific CPD. The National Centre encourages schools and colleges to learn from their own best practice through collaboration among staff and by sharing good practice locally, regionally and nationally. This collaboration takes place virtually through the NCETM portal and 'face to face' through a network of Regional Coordinators in nine regions throughout England.

In the remaining countries where teacher collaboration is centrally supported, it is mainly done through websites, virtual learning platforms, blogs or other types of social networking sites which target teachers of all subject areas, including mathematics.

([7]) http://www.primas-project.eu

([8]) A list of all centrally promoted activities fostering teacher cooperation and collaboration can be found in the Annex.

For example, in **Bulgaria**, a network of innovative teachers has been established. Within this network registered users share electronic learning content, inform each other about good practices in the learning process, communicate with other members and create blogs where they can establish a personal profile and present their work.

A similar website in **Denmark** is entitled 'Educational Meeting Universe'. It provides teachers with a broad range of teaching resources for each subject, including mathematics. Teachers can also suggest teaching materials themselves and exchange information.

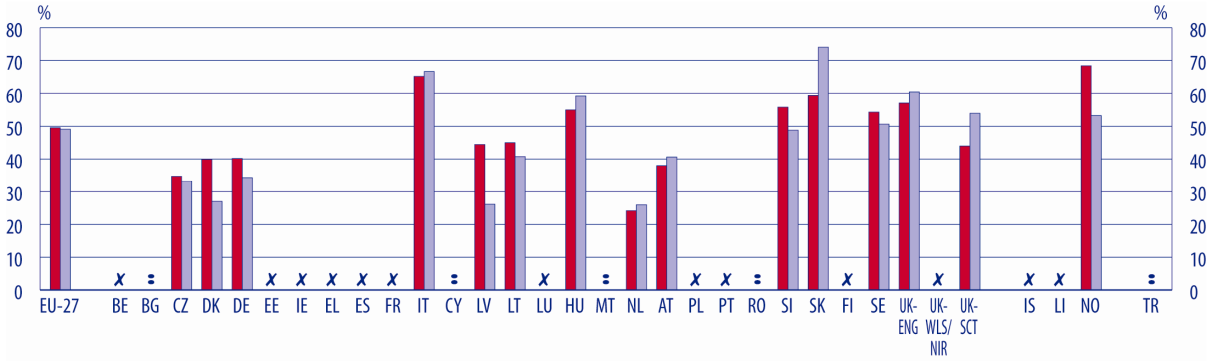
In the **United Kingdom** (**Scotland**), a major support for teachers is 'Glow' – a national intranet for education that enables every teacher in Scotland to access it and use the facility to communicate with any other teacher in Scotland through a range of open forum facilities or through video-conferencing. The system also allows any teacher to upload work, ideas or other documents that can then be shared nationally. The national glow groups for mathematics and numeracy also contain notes on forthcoming events, national and international developments and links to web sites that have been highlighted as useful.

The TIMSS 2007 international survey has, likewise, investigated collaboration between teachers. Figure 6.7 below presents two aspects of collaboration, namely discussions between teachers about how to teach a particular concept and working on teaching materials with other staff. Moreover, it has directed this question at generalist teachers in the fourth grade who teach a range of subjects, including mathematics, and specialist teachers of mathematics in the eighth grade.

The data reveals that, on average in the participating European countries, around 50 % of students in the fourth grade have teachers who report having had discussions about teaching particular concepts and preparing instructional materials together with other teachers between one and three times per week or daily, or almost daily. The proportion ranges between around 25 % of pupils in the Netherlands and around 65 % in Italy with teachers engaging in both type of collaborative activity. In the eighth grade, on the other hand, which involves mathematics teachers in particular, the averages are lower for both types of collaborative activities. The lowest rates for pupils whose teachers collaborated on both activities can be found in the Czech Republic (14.3 % – discussions on concepts, and 22.7 % – preparing teaching materials), and the highest rates were in Cyprus (more than 60 % in both areas).

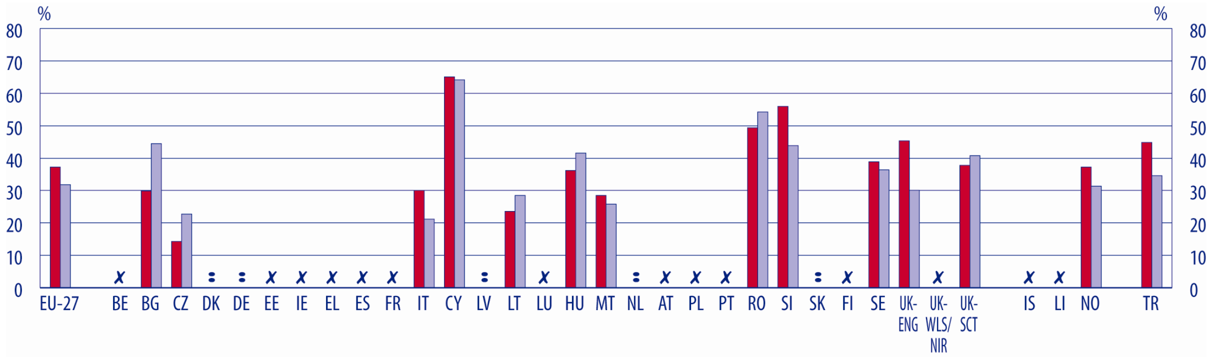
Figure 6.7: Collaboration (at least once per week) between teachers on the teaching process or on the development of teaching materials at primary and secondary level (ISCED levels 1 and 2), 2007

4th grade



	EU-27	BG	CZ	DK	DE	IT	CY	LV	LT	HU	MT	NL	AT	RO	SI	SK	SE	UK-ENG	UK-SCT	NO	TR
Discussions about how to teach a particular concept	49.6	:	34.7	39.9	40.1	65.2	:	44.4	45.0	55.0	:	24.1	38.0	:	55.8	59.4	54.3	57.1	44.0	68.4	:
Working on teaching materials	49.1	:	33.2	27.0	34.3	66.7	:	26.1	40.7	59.2	:	25.9	40.6	:	48.8	74.0	50.6	60.5	54.0	53.2	:

8th grade



Discussions about how to teach a particular concept
Working on teaching materials
x Countries not participating in the survey

	EU-27	BG	CZ	DK	DE	IT	CY	LV	LT	HU	MT	NL	AT	RO	SI	SK	SE	UK-ENG	UK-SCT	NO	TR
Discussions about how to teach a particular concept	37.3	29.8	14.3	:	:	30.0	65.1	:	23.5	36.3	28.4	:	x	49.4	56.0	:	38.9	45.4	37.9	37.3	44.9
Working on teaching materials	31.7	44.5	22.7	:	:	21.1	64.2	:	28.4	41.6	25.7	:	x	54.3	43.9	:	36.5	30.0	40.8	31.3	34.7

Source: IEA, TIMSS 2007 database.

Explanatory note

The Figure shows TIMMS results for the percentage of fourth and eighth grade students whose mathematics teachers indicated that they had had interactions with other teachers in the form of discussions about how to teach a particular concept or working on the preparation of teaching materials 'daily, or almost daily', or '1-3 times per week'. The answer categories included in the TIMMS questionnaire, but omitted here, were '2 or 3 times per month' and 'Never or almost never'.

6.3.2. School management support

The context in which teachers work and collaborate is formed, in part, by the general conditions in their workplace, particularly important is the support offered by the head of their school. School principals can create a supportive environment for teachers' continuing learning by creating a positive climate in the school. This view is in line with the findings on the important role the general school climate plays in changing teachers' practice and improving student learning (European Commission, 2010).

In the context of mathematics teaching, the status of the subject within schools impacts on the ability of teachers to convey its importance, applicability, etc. Conversely, a school environment that fails to provide the necessary infrastructure for quality teaching, such as support from the school principal, time, space and other resources, can thwart the best competences, attitudes and efforts of mathematics teachers (Krainer, 2006).

Such findings lead to the assumption that initiatives or programmes to develop school heads' understanding and role in supporting mathematics teaching in their schools may also contribute to supporting mathematics teachers in their work. However, these types of programme are centrally advocated in only a minority of European countries – Belgium (French Community), Germany, France, Malta, the Netherlands, Slovenia, and Turkey.

> To give an example, in **Slovenia**, there is one programme related to the National Assessment in Mathematics. The aim of this programme is to train head teachers to analyse, together with the mathematics teacher, assessment results in mathematics for pupils from different schools. The purpose of the analysis is to help the school reflect on the effectiveness of its teaching in comparison with other schools and to develop ways to improve mathematics lessons for students.
>
> In **Turkey**, following the development of the new curriculum, principals (as well as teachers and inspectors) are invited to in-service education programmes organised by the Ministry of National Education where staff are introduced to the new curriculum, up-to-date teaching techniques, new developments in educational technology, etc.

6.4. Initial education for mathematics/science teachers: generalist and specialist programmes – SITEP results

6.4.1. Introduction and methodology

As previously discussed, teacher education is recognised as an important factor for ensuring high teaching standards and positive educational outcomes. The previous sections in this chapter presented an overview of central regulations, recommendations and guidelines related to the structure and content of programmes for mathematics teacher education. However, in many European education systems, higher education institutions have a high level of autonomy in determining the content of teacher education programmes. Furthermore, it is important to examine the extent to which regulations or recommendations are implemented. For these reasons, the Eurydice unit at EACEA developed a new European-level Survey on Initial Teacher Education Programmes in Mathematics and Science (SITEP).

The objective of the survey was to gain information on the content of teacher education programmes that goes beyond the recommendations given by the authorities responsible for higher education in each country. The survey also aimed to show how specific competences and skills, which are considered crucial for future mathematics and science teachers, are taught within existing initial teacher education programmes and how they are integrated in the overall workload.

The survey was targeted at 815 higher education institutions across Europe that provide 2 225 initial teacher education programmes for primary and/or lower secondary general education teachers. In each country, the programmes were analysed in accordance with the national qualification framework and the specific criteria that apply to the level and minimum length of initial teacher education. Alternative pathways to become a teacher (short professional courses for side entrants from other professions) were excluded from the framework as they follow different regulations and are only available in some countries.

The development of the SITEP theoretical framework started at the beginning of 2010 and a comprehensive list of institutions providing initial teacher education was prepared. In September 2010, a consultation was organised to validate and test the draft questionnaire with the Eurydice national units, researchers and policymakers. Consequently, the final version of the questionnaire was developed and 22 linguistic versions were prepared taking into consideration country specific terms and interpretations. The data collection was carried out between March and June 2011.

The survey used an online data collection tool. Responses were received from 205 institutions offering 286 programmes. As the response rates and/or the number of responses by country were usually low, the following sections present only pooled results from the education systems with the highest response rates, namely Belgium (Flemish Community), the Czech Republic, Denmark, Germany, Spain, Latvia, Luxembourg, Hungary, Malta, Austria and the United Kingdom (a total of 203 teacher education programmes). The exact response rates by country can be found in the Annex 3.

Due to low response rates, the data are not fully representative and therefore should be considered as an indication only. Reporting by country or even presenting standard errors was not meaningful.

6.4.2. General description of education programmes for generalist teachers and specialist mathematics/science teachers

SITEP addressed two distinct types of teacher initial education, namely programmes for generalist teachers and programmes for specialist teachers. A generalist teacher is defined as a teacher who is qualified to teach all, or almost all, subjects or subject areas in the curriculum. A specialist teacher is a teacher qualified to teach one or two different subjects. SITEP was directed at only specialist teacher education programmes for mathematics or natural sciences.

The descriptive analysis of the SITEP results seems to reflect the common pattern of what was already known about initial education programmes for generalist and specialist teachers (see Figure 6.8). As expected, the generalist teacher programmes usually award a Bachelor's degree, while specialist mathematics/science teacher education programmes were organised at Master's or equivalent level. Correspondingly, the mean duration of the generalist teacher education programmes was longer than that of the specialist teacher education programmes. However, it is important to note that access to Master's degree programmes is usually conditional on graduation from a Bachelor's degree or equivalent programme. This leads to an overall length of study for specialist teachers to 4-6 years ([9]). The generalist teacher education programmes usually produced graduates qualified to teach at primary or pre-primary levels of education, while most specialist mathematics/science teacher education programmes were preparing graduates to teach at lower and upper secondary levels. Predictably, the proportion of female graduates was higher in generalist teacher education programmes than in specialist programmes for mathematics/science teachers.

([9]) For more information on the minimum length of initial teacher education for general lower secondary level, see EACEA/Eurydice, Eurostat (2009), p. 155.

Teacher education programmes for both generalist and specialist teachers are normally delivered either by a single unit/department or by a combination of units/departments in a faculty or institution. The latter model is more common with respect to specialist teacher education.

Figure 6.8: Some descriptive statistics of mathematics and science teachers training programmes, 2010/11

	Generalist		Specialist	
	COUNT	PER CENT	COUNT	PER CENT
Number of programmes surveyed	43	-	160	-
Awarded qualification – Bachelor's degree or equivalent	38	88.4	43	26.9
Awarded qualification – Master's degree or equivalent	3	7.0	75	46.9
Mean duration of the programme (years)	3.7	-	2.6	-
Qualifies for teaching at pre-primary level	17	39.5	6	3.8
Qualifies for teaching at primary level	33	76.7	30	18.8
Qualifies for teaching at lower secondary level	6	14.0	138	86.3
Qualifies for teaching at upper secondary level	3	7.0	106	66.3
Average proportion of female students	-	60.3	-	55.7

Source: Eurydice, SITEP survey.

Explanatory note

As institutions can provide teacher qualifications for more than one level of education, the percentages may therefore not add up to 100 %.

As the response rates were low, the data are not representative and therefore should be considered as an indication only.

Despite low response rates, the general characteristics of teacher education programmes that answered the SITEP survey correspond to the usual features or distinctions between generalist and specialist teachers. Therefore, some further analysis of the pooled results was performed.

6.4.3. Knowledge and competences in initial teacher education programmes for generalist and specialist mathematics/science teachers

The main focus of SITEP was the analysis of the specific competences or content areas covered during the initial education of teachers of mathematics/science. Additional information was gathered on how the competences were addressed in the programmes. The response categories offered made a distinction between 'general references'; competences/content included as 'part of a specific course' and competences/content 'included in assessment'. In order to facilitate direct comparisons, the three types of responses were assigned a different weight. It was assumed that the least attention to a competence/content area was given when only a general reference was made in the programme (one point). Medium weight (two points) was attributed when the competence/content was included in a specific course, and the highest weight was given when the competence was included in assessment (three points). If more than one answer option was chosen, the highest value was assigned. Figure 6.9 lists the responses as percentages by category and as total using weighting.

The survey aimed to gather information about certain competences and skills that, according to the scientific literature, are crucial for future mathematics or science teachers (see the list in Figure 6.9). Most of the competences and content areas analysed were grouped into several broader categories. Only one competence, namely 'knowing and being able to teach the official mathematics/ science curriculum' stood alone. The official mathematics/science curriculum is a formal document that describes the objectives and content of mathematics/science courses, as well as the teaching, learning, and assessment materials available. Knowledge of the curriculum therefore could be seen as an overarching competence and is analysed separately. However, other competences were grouped into three broader categories.

The largest category included six competences or content areas related to innovative teaching and assessment approaches. It contained the application of inquiry- or problem-based learning, collaborative learning, portfolio assessment and the use of ICT (previously discussed in Chapters 2 and 3). Two competences in this category may require additional explanation. Personalised teaching and learning means taking a highly structured and responsive approach to each child's or young person's learning, so that all students are able to progress, achieve and participate. It means strengthening the link between learning and teaching by engaging pupils – and their parents – as partners in learning. In addition, the category includes one competence that is related to an understanding of the production of scientific knowledge. The competence 'explaining the social/cultural aspects of mathematics/science' refers to a way of thinking that conceives knowledge production as a social practice that is dependent on the political, social, historical and cultural realities of the time. It includes examining and being able to explain the values implicit in scientific practices and knowledge; looking at the social conditions as well as the consequences of scientific knowledge and its changes; and studying the structure and process of scientific activity.

Another distinct category included five competences summarised under a heading 'dealing with diversity'. It included two types of competences: those related to being able to teach pupils with different abilities and interests, and those that promote sensitivity to gender issues. As discussed earlier (see Chapters 4 and 5), this type of competence is important in addressing the issues of low achievement, challenging gifted students and motivating both girls and boys.

Lastly, three competences were put together into the 'collaboration with peers and research' category. It includes important aspects of teachers' work, such as conducting and applying research, as well as collaborating with colleagues on pedagogy and innovative teaching approaches.

As answers in each of the categories were interlinked and had consistent patterns ([10]), it was possible to compute the scale totals. Figure 6.9 lists the scale averages per item in order to account for different numbers of questions in each category.

Generalist teacher education programmes and mathematics/science teacher education programmes were rather similar in the ways they addressed mathematics/science competences and content areas. On average, all competences/content areas were given medium importance, analogous to the category 'part of a specific course' (see Figure 6.9).

([10]) The Cronbach alpha coefficients indicated sufficient internal consistency of the scales. 'Creating a rich spectrum of teaching situations and assessment' had Cronbach alpha=0.68, 'dealing with diversity' had Cronbach alpha=0.75 and 'collaboration with peers and research' had Cronbach alpha=0.67. Cronbach's alpha is the most widely used index of the reliability or the internal consistency of a scale, which is based on the average of all inter-item correlations in a survey instrument (for explanation, see Cronbach (1951), Streiner (2003).

Figure 6.9: Addressing knowledge and competences in teacher education programmes for generalist and specialist mathematics and science teachers, percentages and total weightings, 2010/11

	General reference %	Part of a specific course %	Included in assessment %	Not included %	Total
Generalist teachers					
Knowing and being able to teach the official mathematics/science curriculum	46.5	83.7	76.7	0.0	**2.7**
Creating a rich spectrum of teaching situations					**2.1**
Applying inquiry-based or problem-based learning	51.2	72.1	65.1	2.3	2.4
Applying collaborative or project-based learning	48.8	62.8	62.8	4.7	2.3
Using ICT for teaching mathematics/science phenomena through simulations	34.9	76.7	55.8	7.0	2.3
Explaining the social/cultural aspects of mathematics/science	44.2	69.8	46.5	2.3	2.2
Applying personalised learning techniques	51.2	44.2	32.6	11.6	1.8
Applying portfolio-based pupil assessment	37.2	41.9	25.6	32.6	1.4
Dealing with diversity					**1.6**
Teaching a diverse range of pupils with different abilities and motivation to study mathematics/science	44.2	58.1	39.5	11.6	2.0
Using diagnostic tools for early detection of pupils' learning difficulties in mathematics/science	39.5	58.1	37.2	23.3	1.8
Analysing pupils' beliefs and attitudes towards mathematics/science	46.5	58.1	23.3	14.0	1.7
Avoiding gender stereotypes when interacting with pupils	55.8	34.9	23.3	20.9	1.4
Teaching mathematics/science taking into account the different interests of boys and girls	32.6	37.2	25.6	32.6	1.3
Collaboration with peers and research					**1.9**
Applying research findings to daily teaching practice	62.8	62.8	34.9	7.0	2.0
Collaborating with colleagues on pedagogy and innovative teaching approaches	53.5	53.5	34.9	18.6	1.8
Conducting pedagogical research	37.2	58.1	37.2	20.9	1.8
All competences					**1.9**
Specialist teachers					
Knowing and being able to teach the official mathematics/science curriculum	21.9	83.1	61.3	2.5	**2.5**
Creating a rich spectrum of teaching situations					**2.1**
Applying collaborative or project-based learning	24.4	76.3	49.4	1.9	2.4
Applying inquiry-based or problem-based learning	25.0	78.8	46.3	4.4	2.3
Using ICT for teaching mathematics/science phenomena through simulations	21.3	76.9	44.4	6.9	2.2
Explaining the social/cultural aspects of mathematics/science	31.3	70.6	29.4	6.9	2.0
Applying personalised learning techniques	35.0	63.8	36.9	8.8	2.0
Applying portfolio-based pupil assessment	30.6	47.5	22.5	24.4	1.5
Dealing with diversity					**1.8**
Teaching a diverse range of pupils with different abilities and motivation to study mathematics/science	26.9	73.1	46.9	4.4	2.3
Using diagnostic tools for early detection of pupils' learning difficulties in mathematics/science	27.5	61.9	31.3	15.0	1.8
Avoiding gender stereotypes when interacting with pupils	42.5	52.5	20.6	10.0	1.7
Teaching mathematics/science taking into account the different interests of boys and girls	36.9	50.0	25.0	18.1	1.6
Analysing pupils' beliefs and attitudes towards mathematics/science	35.0	48.8	18.1	15.0	1.6
Collaboration with peers and research					**2.0**
Applying research findings in daily teaching practice	36.3	65.0	40.6	4.4	2.1
Collaborating with colleagues on pedagogy and innovative teaching approaches	33.1	66.9	33.8	5.0	2.0
Conducting pedagogical research	28.8	56.3	39.4	18.1	1.9
All competences					**2.0**

Source: Eurydice, SITEP survey.

Explanatory note

The columns 'General reference', 'Part of specific course', 'Included in assessment', 'Not included' show the percentage of total programmes which include these elements. As the respondents could choose more than one option, the sum of the percentages may exceed 100 %. The column 'Total' shows the average highest score for a competence/content area, where General reference = 1; Part of specific course = 2; 'Included in assessment' = 3; 'Not included' = 0. Scale total shows average per scale item.

As the response rates were low, the data are not representative and therefore should be considered as an indication only.

Knowing and being able to teach the official mathematics/science curriculum

The overarching competence 'knowing and being able to teach the official mathematics/science curriculum' was the most important competence emphasised in both generalist and specialist teacher education programmes. Knowledge of the curriculum was assessed in 76.6 % of the examined generalist teacher education programmes and 61.3 % of the mathematics/science teacher programmes. Moreover, all generalist teacher education programmes addressed the knowledge of mathematics/science curriculum at least as a general reference.

Creating a rich spectrum of teaching situations

The scale 'creating a rich spectrum of teaching situations' was often addressed in the programmes provided by the institutions that answered the SITEP survey. This type of competence was mostly 'part of a specific course' (scale average for both generalist and specialist teachers was 2.1 points).

Collaborative learning, or making pupils work together in small groups on one or more phases of a task, is an important motivational aspect in learning (see Chapter 5). According to the research, project work with no known answer or no previously learned solution should become an essential educational activity in science and mathematics involving experiments or construction of models (see Chapter 2). The responses to SITEP showed that these innovative forms of learning were often addressed when training prospective teachers. 'Applying collaborative or project-based learning' was included in the assessment in 62.8 % of generalist teacher programmes and in 49.4 % of mathematics/science teacher education programmes. It was 'part of a specific course' in 62.8 % of generalist teacher programmes and in 76.3 % of specialist teacher education programmes.

Inquiry-based and problem-based learning is currently widely advocated for science and mathematics teaching as a way to increase motivation and achievement. These forms of pupil-centred and self-directed learning were usually addressed as 'part of a specific course'. 'Applying inquiry-based or problem-based learning' was 'part of a specific course' in 72.1 % of generalist programmes and 78.8 % of specialist teacher programmes.

Using ICT for teaching mathematics/science phenomena through simulations was also widely addressed in generalist and specialist teacher education. Simulation is understood here as a computer program that attempts to simulate an abstract model of a particular system. Use of ICT for teaching through simulations was included in 'part of a specific course' in more than 70 % of generalist and specialist teacher education programmes.

One competence, namely 'applying portfolio-based pupil assessment', stood out from the category 'creating a rich spectrum of teaching situations' with lower values than other items. Portfolio assessment was not addressed at all in about a third of the generalist teacher education programmes and in about a quarter of mathematics/science teacher education programmes. However, the prospective teachers were themselves often assessed using portfolio evaluation (see the discussion below, Figure 6.12), which might prepare them to use this type of assessment in their teaching. These results might indicate that innovative forms of assessment are practiced, but not explicitly discussed during teacher education.

Collaboration with peers and research

The other two competence categories were given somewhat less attention in the teacher education programmes that answered the SITEP survey. The category 'collaboration with peers and research' had an average importance in programmes for specialist and generalist teachers. 'Collaborating with colleagues on pedagogy and innovative teaching approaches' and 'conducting pedagogical research' were not addressed in about a fifth of generalist teacher programmes. Collaboration with colleagues was included as part of a specific course in two-thirds of mathematics/science teacher programmes while conducting pedagogical research was not addressed in a fifth of all programmes.

Dealing with diversity

Meeting the needs of a diverse range of students and the different interests of boys and girls are important for motivating students to learn (see more Chapter 5). However, 'dealing with diversity' was the least addressed competence in both the generalist and specialist teacher education programmes according to the survey responses received. In particular, competences relating to dealing with diversity and gender were less frequently addressed in generalist teacher education programmes than in specialist. Such findings might be a reflection of current national policies on gender in education, as gender-sensitive teaching is promoted in only one-third of European countries (EACEA/Eurydice 2010, pp. 57-59).

The findings of the survey generally reinforce the information reported by national authorities. Central-level documents usually mention that teachers should know how to teach the mathematics curriculum and how to create a variety of teaching and learning situations. Specific assessment methods or teaching in a gender sensitive way are less often emphasised.

6.4.4. Patterns in addressing competences/content in teacher education programmes

After examining the overall importance attributed to specific competences in the teacher education institutions which responded to the survey, we considered whether there were any significant patterns in the way programmes addressed these competences. This section therefore analyses whether any programmes systematically gave priority to some categories of competences over others, or whether there were groups of teacher training programmes addressing the competences in particular ways.

For these purposes, the teacher education programmes examined were classified according to the scale averages (mean) for the various categories of competences: 'creating a rich spectrum of teaching situations', 'dealing with diversity' and 'collaboration with peers and research'; and the specific competence 'knowing and being able to teach the official mathematics/science curriculum'. The responses revealed four distinct groups, or clusters, where the programmes in the same cluster addressed the competences in a similar way (see Figure 6.10) ([11]).

Two of the four groups of teacher education programmes were extreme opposites. At the top end of the scale, one cluster had the highest values in all the competences analysed and virtually all programmes in this cluster assessed prospective teachers in their knowledge of the curriculum. The other competences analysed were also usually assessed in this cluster and relatively few competences fell into the lower value response groups. Approximately one fifth of the programmes that answered the survey belonged to this cluster.

([11]) A disjoint cluster analysis was performed on the basis of the analysed competences/content scales. 4-cluster solution explained 63 % of the total variance. 5-cluster model explained only 3.8 % additional variance, while 3-cluster solution decreased the explained variance by 13 %.

Figure 6.10: Means of the competences/content scales and distribution of teacher education programmes, by clusters, 2010/11

	Clusters			
	High values	High/medium except diversity	Medium	Low values
Knowing and being able to teach the official mathematics/ science curriculum	3.0	2.8	2.4	2.0
Creating a rich spectrum of teaching situations	2.7	2.3	1.7	1.4
Dealing with diversity	2.6	1.4	2.0	1.0
Collaboration with peers and research	2.7	2.0	1.8	1.3
All teacher education programmes	22.7 %	33.0 %	26.1 %	18.2 %
Generalist teacher education programmes	25.6 %	34.9 %	14.0 %	25.6 %
Specialist teacher education programmes	21.9 %	32.5 %	29.4 %	16.3 %

Source: Eurydice, SITEP survey.

Explanatory note

As the response rates were low, the data are not representative and therefore should be considered as an indication only.

The cluster at the other end of the scale had the lowest values in all competences analysed. On average, knowledge of the curriculum in programmes belonging to this cluster was included as 'part of a specific course'. Some of the programmes in this cluster included knowledge of the curriculum in their assessment of prospective teachers, but a few did not mention this competence at all or only made a general reference to it. This group included teacher education programmes that either did not refer at all to some of the analysed competences, or made only a general reference to most of them. More than half of the programmes in this cluster did not include any of the competences in question in their assessment process. In addition, dealing with diversity issues was usually either not mentioned, or mentioned only as a general reference in these programmes. 18.2 % of the programmes that answered the SITEP belonged to this cluster with low values in all dimensions.

Obviously, the other two clusters were somewhere in-between these two extremes. The second cluster had the second highest values in all competence areas except diversity issues and was labelled 'high/medium except diversity'. It included about one third of the programmes analysed. The third cluster, which included 26.1 % of the programmes analysed, had the second highest values on 'dealing with diversity' scale, and the third highest on all the other scales. It was labelled 'medium'.

Interestingly, there were only minor differences between generalist and specialist teacher education programmes. There were very similar proportions of generalist and specialist teacher programmes in the cluster with high values in all dimensions as well as in the cluster with high/medium values in all dimensions except diversity. In the third cluster (with higher values for diversity issues), there were proportionally more specialist teacher programmes than generalist teacher programmes; while in the fourth cluster (with the lowest values on all competences) there were more generalist teacher programmes.

These results suggest that there seems to be a tendency to treat the majority of competences in a similar way throughout a given programme. For example, if one category is included in the assessment process, it is likely that the rest will be also. If a major competence category is just mentioned as a general reference, the others are not likely to receive greater attention. There are, however, a few exceptions. Knowledge of the curriculum stands out from this tendency, as reference to the curriculum is made in virtually all programmes and the majority of them also include this in the assessment of prospective teachers. In addition, about a third of the teacher education programmes analysed place quite high emphasis on all dimensions except diversity issues. In general, dealing with different levels of achievement and sensitivity to gender issues seems to be inadequately addressed in many teacher education programmes.

The SITEP survey also included a few specific questions on some other important aspects of teacher education programmes. Partnerships with external stakeholders and assessment in teacher education programmes are briefly discussed in the next sections.

6.4.5. Partnerships between teacher education providers and external stakeholders

The providers of generalist and specialist teacher education programmes which responded to the survey gave very similar answers regarding collaboration with external stakeholders (see Figure 6.11). The main partners of teacher education institutions were primary and secondary schools. There was cooperation between the majority of both generalist and specialist teacher education programmes and schools in the area of programme implementation. Naturally, teacher education programmes cooperate with schools in organising in-school placements. Moreover, schools were also the main partners in the development of programme content and research.

Figure 6.11: Teacher education institutions' involvement in partnerships/collaborations, for generalist and specialist teachers (mathematics/science), 2010/11

	Programme content		Programme implementation		Research	
	Generalist	Specialist	Generalist	Specialist	Generalist	Specialist
Primary or secondary schools	53.5	46.3	76.7	85.0	23.3	22.5
National or local government organisations	44.2	40.6	46.5	50.0	9.3	11.3
Companies	2.3	2.5	9.3	6.9	7.0	5.6
Civil society organisations	7.0	10.0	18.6	20.0	14.0	13.8

Source: Eurydice, SITEP survey.

Explanatory note

As the response rates were low, the data are not representative and therefore should be considered as an indication only.

The responses of approximately half of the teacher education programmes indicated that collaboration existed with national or local government organisations in the area of programme implementation. Slightly fewer programmes had set up collaborative activities or projects with government organisations regarding programme content. Very few had established partnerships with civil society organisations and companies.

Interestingly, teacher education institutions collaborated less with external stakeholders over research matters than in any other area. Only 20 % of teacher education programmes reported that they used partnerships with schools for carrying out research. Therefore, there seems to be further opportunities for collaborating with external stakeholders on research and development into innovative teaching approaches for educating future teachers.

6.4.6. Assessment of generalist and specialist teachers

Assessment is an important part of the teaching and learning process which can take different forms and serve different functions. Therefore, the question on assessment in teacher education programmes addressed both content knowledge and teaching skills (see Figure 6.12). The most common way of assessing content knowledge in both generalist and specialist teacher education programmes was through written and oral tests; while observation of teaching practice was most usually used to assess teaching skills.

Portfolio evaluation was the least common form of assessment used with respect to content knowledge, but was used in 58.1 % generalist and 66.9 % of specialist teacher education programmes to assess teaching skills. This is quite an encouraging result, as portfolio evaluation is a non-traditional (or innovative) form of assessment, which according to Collins (1992, p. 453) is 'a container of collected evidence with a purpose' that helps to increase students' responsibility for their own learning.

Figure 6.12: Assessment of generalist and specialist teachers in mathematics and science teacher education programmes, 2010/11

	Content knowledge		Teaching skills	
	Generalist	Specialist	Generalist	Specialist
Written and oral tests	95.3	86.9	69.8	55.0
Portfolio evaluation	39.5	44.4	58.1	66.9
Observation of teaching practice	48.8	47.5	83.7	91.9
Writing research papers	51.2	56.9	44.2	49.4
Thesis	44.2	61.9	25.6	51.9
Other	62.8	46.3	51.2	46.9

Source: Eurydice, SITEP survey.

Explanatory note

More than one answer category was allowed; therefore the percentages do not make 100.

As the response rates were low, the data are not representative and therefore should be considered as an indication only.

However, there were some differences between generalist and specialist teacher education programmes. Even though writing research papers was often used in both types of programme, a thesis was a much more common form of assessment in specialist than in generalist teacher education programmes. For assessing content knowledge, the thesis was used in 44.2 % of the generalist teacher programmes and 61.9 % of the specialist mathematics/science teacher education programmes examined.

This section of the study has attempted to give some indication of how future teachers are trained today in number of European education countries. It must be borne in mind, however, that this analysis of the content and skills taught and forms of assessment used in both generalist and specialist teacher education programmes only provides a guide to the knowledge and skills expected of European teachers, their actual knowledge and practical ability to teach in the classroom cannot be directly inferred from the content of teacher education programmes.

Summary

This review of the current state of the mathematics teaching profession in Europe and the policies and practices relating to teachers' initial education and professional development has revealed several positive trends as well as some areas where improvements could be made.

Some European countries seem to be concerned about the unbalanced age profile of mathematics teachers. International survey data from TIMSS confirms these fears to a certain extent, particularly for Bulgaria, Germany, Italy and Romania. However, looking at the wider evidence, European statistical data suggest that the ageing of the teaching workforce may reflect a general trend in many countries, irrespective of the subject area. More detailed analysis is necessary to examine the extent of the problem and determine appropriate policy solutions, whether initiatives should target teachers belonging to a particular discipline such as mathematics, or whether more global issues should be addressed, such as the level of financial investment in the teaching profession, including whether new incentives are needed to attract and retain teachers.

With respect to the gender balance in the European teaching body, a high percentage of female teachers can be found at primary level in all subjects, including mathematics. Only Denmark seems to have achieved a more equal distribution of male and female teachers. At lower secondary level, the preliminary data presented here hints at a more balanced proportion of male and female mathematics teachers.

Countries share a number of challenges regarding the supply of qualified mathematics teachers. There seems to be a lack of mathematics teachers at secondary level in some countries, which is confirmed by the 2009 PISA results, particularly for Luxembourg and Turkey. But there are also problems at primary level where the generalist teachers who are responsible for mathematics reportedly lack deeper mathematics subject knowledge. In the majority of countries, where central level regulations or recommendations concerning ITE identify a minimum proportion of the entire course load to be dedicated to developing prospective teachers' subject knowledge of mathematics, the percentages are disproportionately higher for specialist (and semi-specialist) mathematics teachers than for generalist teachers. Only a few countries so far seem to be taking steps to change this trend by way of reforming teacher education, training or working conditions. The United Kingdom (England) is exceptional in undertaking initiatives targeting the development of specialist knowledge among primary school teachers as well as supporting the development of specialist mathematics teachers at this level.

When it comes to the initial education of prospective mathematics teachers, research evidence highlights the importance of providing them with 'mathematical knowledge for teaching'. Across Europe, in most countries with central level regulations, recommendations and/or guidelines regarding the content of ITE programmes, a wide range of areas of mathematical knowledge are covered. However, the aspect which is least frequently included is the knowledge and understanding of how to teach mathematics in a gender sensitive way.

A large number of countries advise that prospective specialist and semi-specialist mathematics teachers are systematically assessed in their mathematics teaching skills and that prospective generalist teachers teaching mathematics are assessed on their mathematics subject knowledge. The latter is the case not only during or at the end of their study programmes, but also at the beginning of it in the form of an entrance examination. Higher education institutions implementing the ITE programmes are usually responsible for the content, form and evaluation of these examinations. Central exams for prospective mathematics teachers are rare in Europe.

Interestingly, the EACEA/Eurydice pilot survey of teacher education programmes (SITEP) revealed more similarities than differences between generalist and specialist teachers. The most important competence addressed in both types of teacher education is the knowledge and ability to teach the official mathematics/science curriculum. It is very often included in the assessment of prospective teachers. Creating a rich spectrum of teaching situations, or applying various teaching techniques, is usually a part of a specific course in both generalist and specialist teacher education programmes. Applying collaborative or project-based learning and inquiry- or problem-based learning is frequently addressed in both types of teacher education programmes. Dealing with diversity, i.e. teaching a diverse range of students, taking into account different interests of boys and girls, and avoiding gender stereotypes when interacting with students, is less often addressed in generalist teacher education programmes than in programmes that prepare mathematics/science teachers. Generally, these competences are the least often addressed in both types of programme, although diversity issues are important in order to improve motivation and tackle low achievement.

The academic literature suggests that professional development for mathematics teachers should be subject-based and collaborative. European countries cover a wide range of topics related specifically to mathematics teaching through centrally promoted CPD programmes. However, TIMSS 2007 data showed that participation rates were rather low, particularly at primary level where only around one third of students, on average in the EU, had teachers who had attended professional development courses on topics such as teaching the mathematics curriculum, developing students' problem-solving skills or integrating ICT into mathematics teaching in the previous two years. Only a minority of European countries offer real incentives, financial or otherwise, to promote teacher participation in professional training on new methods and approaches to mathematics teaching.

Topics for mathematics teachers' professional development that are the least frequently advocated at a central level include the use of research and research methods in daily teaching practice – even though the importance of this issue is emphasised in a large body of research – and again, the gender sensitive teaching dimension in mathematics is not often highlighted.

On the other hand, a majority of European countries recognise the importance of cooperation and collaboration between mathematics teachers (and other relevant experts) for their professional development and therefore advocate or provide actual support for teachers' networks where ideas and experiences can be exchanged, and teaching approaches, methods and materials can be shared. These can take the form of projects, conferences or meetings, or virtually through websites, blogs or other social networking sites.

Finally, in-service programmes for school heads to support the work of their mathematics teachers and encourage collaboration exist only in a minority of countries. Such programmes could help raise the general status of mathematics within schools and have a positive impact on the ability of teachers to convey the importance of the discipline.

CONCLUSIONS

Mathematics is recognised as being a subject of great importance both within school and in wider society. Its concepts and processes are essential in a wide range of disciplines, professions and areas of life. The latest results of international surveys like PISA and TIMSS show that over the years several countries have been successful in improving their students' knowledge and skills in mathematics, and some have managed to narrow the gap between high and low achieving students. However, across Europe, there still remain a large proportion of students who do not reach the expected level of mathematical literacy.

The research reviewed in this report points to ways in which mathematics teaching can help improve student performance and engagement; it also highlights some of the contextual factors which influence mathematics learning. In addition, the study has examined the wide range of policies and practices that shape mathematics education in European countries. These conclusions present the key findings of this report and highlight the areas that would benefit from further research or developments in policy in order to bring about an improvement in mathematics learning outcomes.

A. Translating the revised mathematics curricula into classroom practice

The mathematics curriculum is one of the most important steering documents that shapes classroom practice. In Europe, mathematics curricula are mostly issued by central education authorities and lay down all the essential learning aims and outcomes for mathematics education. Curricula also give indications of the minimum recommended time that should be dedicated to mathematics teaching – between 15 % and 20 % of the overall taught time in primary, and slightly less at lower secondary education, making it the second most important subject after the language of instruction.

Over the last decade – and most notably since 2007 – the great majority of countries have made revisions to their mathematics curricula to focus more on the competences and skills to be achieved rather than on the content to be covered. In addition, current mathematics curricula have reduced subject content in favour of more cross-curricular links and increased focus on the application of knowledge and problem-solving. The move towards a learning outcomes-based approach is supported by research findings showing that, compared to traditional curricula, outcome-oriented curricula tend to be more comprehensive and flexible. They allow teachers greater autonomy to deliver the set objectives and to be more responsive to the needs of learners; they also contribute to improved student motivation.

The analysis of five areas of competence – mastering basic skills and procedures, understanding mathematical concepts and principles, applying mathematics in real-life contexts, communicating about mathematics and reasoning mathematically – showed that, although they are all mentioned in European countries' curricula, specific teaching and assessment methods for these skills are rarely recommended. Evidence from academic research shows, however, that the effective translation of curriculum objectives into classroom practice depends on a variety of factors: the provision of support for teachers, while also respecting their didactic autonomy, is one important factor, another is the need to align student assessment, and in particular high stakes tests, with new developments in mathematics teaching.

B. Applying a range of teaching approaches to meet the needs of all learners

Across Europe, educational policy seems to be in accordance with findings from research and international surveys regarding approaches to teaching mathematics: there is no one correct way of teaching mathematics, but different methods can be effectively applied in specific contexts and for particular learning outcomes. With the exception of very few countries, most central authorities provide some form of national guidance about teaching approaches in mathematics at primary and secondary level.

Among the methods promoted are problem-based learning, exploration and investigation as well as the use of real life contexts to make mathematics more relevant to students' own experience. International surveys confirm that problem-based learning activities are common in European classrooms. Other more traditional approaches to teaching mathematics such as memorising are rarely prescribed or recommended, although students in a number of countries reported the use of such strategies.

Overall, there is a need to find a balance between methods which foster the students' learning of mathematical knowledge with the development of their mathematical skills. In particular, there is potential for strengthening the support for teaching approaches which promote active learning, critical thinking and students' ability to apply theoretical knowledge to real life situations. These methods have consistently shown to impact positively not only on attainment levels but also on attitudes towards mathematics.

Less conclusive evidence is available about the impact of ICT, calculators, student grouping and homework in the context of mathematics instruction. National guidelines on the use of these approaches are rare, except for the use of ICT, which is prescribed or recommended in all countries. International survey data, on the other hand, show that although computers are widely available, they are not frequently used in mathematics lessons. Mathematics teaching that aims to connect to students' daily life cannot ignore technology. However, more research and clear evidence about the benefits of ICT in learning mathematics will be needed to guide its use and enable it to be applied effectively.

C. Effective use of assessment methods: additional support needed for teachers

Student assessment is viewed as a crucial element in the teaching and learning process. It can also play a central role in implementing curriculum reform, since what is taught in schools is often determined by what is assessed. Mathematics is one of the main focuses of national tests in compulsory education as well as in school leaving examinations at the end of upper secondary education. National test results are reportedly used to inform curriculum development as well as teacher training and professional development. However, national information also suggests that they could be used in a more systematic way by policy-makers at the different levels of decision-making.

This report has found that practical guidelines for classroom assessment – particularly guidelines to encourage the use of more innovative forms, such as project-based, portfolio, ICT or self/peer-based assessment – are issued by very few central authorities. Research evidence highlights the importance of classroom assessment in mathematics and the key role that teachers play in preparing and administering it; their role in providing relevant feedback is considered particularly important. National evidence from this study indicates a potential need for additional guidelines as well as other support measures for teachers in using assessment tools.

D. Tackling low achievement: the need to set up targets and monitor the effectiveness of support programmes

The significant proportion of students who do not have basic skills in mathematics is a concern in Europe. In some countries the rates of under-achieving 15-year olds are especially worrying. The essential first steps needed to address the issue at national level include setting up mechanisms to monitor achievement levels, identifying the causes of low achievement in mathematics, and evaluating the effectiveness of support programmes. However, only a few European countries have set national objectives to reduce low achievement in mathematics. Less than half of European countries conduct surveys or have reported on the causes of low achievement in this subject. Even less common are recent evaluations of support programmes for low achievers.

Where reports do exist, they link underachievement in mathematics with factors such as low levels of education among parents, a lack of educational resources and assistance at home, low intrinsic student motivation and inadequate qualifications among teachers. These findings indicate that lowering the proportion of low achievers in mathematics would require a comprehensive approach that simultaneously addresses a range of factors in and out of school.

Research evidence on effective educational measures to tackle low achievement underlines the importance of:

- laying the foundations for mathematics learning as early as pre-primary level;
- providing individual support to tackle difficulties as and when they occur;
- increasing motivation by ensuring that links are established with other subjects;
- making connections with everyday life; and
- involving parents with their children's mathematics education.

The majority of European countries provide national guidelines to address student difficulties in mathematics. Such guidelines are usually broadly formulated recommending, for example, the use of diagnostic tests, curriculum modifications, individual or small-group tuition, and allowing teachers, schools and school providers to choose appropriate forms of support. Targeted programmes such as 'Maths Recovery' in Ireland and the United Kingdom, or similar teaching support that provides practical guidance to teachers and systematic help for students are rare, but they may make an effective contribution to help tackling low achievement in mathematics.

E. Increasing student motivation and engagement through targeted initiatives

Improving student motivation to learn mathematics is important for raising school attainment, for increasing the numbers of students choosing mathematics-related subjects beyond secondary education and for encouraging young people to pursue careers in fields requiring high levels of mathematical knowledge. Results from all the major international surveys, as well as a vast array of academic research, confirm the link between motivation, attitudes, self-confidence and mathematics achievement.

Less than half of European countries have national strategies which seek to improve student motivation in learning mathematics – where these do exist, they are often incorporated within a broader framework which also covers the fields of science and technology. Initiatives that cover all levels of education, from pre-primary to upper secondary, and include a wide range of actions are being implemented only in Austria and Finland. More commonly, countries focus on specific projects, such as support for extra-curricular activities, partnerships with universities and companies, and teaching methods which encourage student engagement. Evaluations of some of these national strategies and activities have shown positive impacts on student motivation, interest and performance in mathematics. The overall effect might be increased, however, if initiatives were targeted specifically at improving student motivation in mathematics, in addition to those which combine mathematics with other disciplines. Furthermore, in addition to the existing programmes that usually target the more able students, attainment might be improved if initiatives had a broader focus on the wider student population but with special measures to target those with low motivation and achievement levels.

Other major concerns in many European countries include the low proportion of MST students – especially female students – compared to other subjects, and skills shortages in areas requiring high levels of mathematical knowledge. Even though studies have shown that the gender gap in attitudes towards mathematics is wider than the actual gap in mathematics achievement, only four countries have launched national activities related to gender issues in schools, while a few others have implemented national campaigns to attract more women into mathematics-related professions. More targeted initiatives are therefore needed to improve levels of motivation and self-confidence among female students in order to increase their participation in areas of study where mathematical knowledge and skills are essential.

F. Widening the teacher's repertoire and encouraging flexibility

As discussed above, teachers play a central role in advancing the reforms in mathematics education. In order to be able to help students develop their mathematical skills, teachers must be able to choose from a wide range of teaching methods, they must be flexible, use different forms of assessment, be able to motivate all types of students and, in particular, empower those with low levels of attainment. To do all this teachers need to be equipped with the necessary knowledge skills and support to respond to the needs of all learners. European countries are currently facing several challenges that must be overcome to reach these goals.

In addition to concerns in some countries about the age and gender profile of mathematics teachers, the biggest challenge seems to be to improve the qualifications of mathematics teaching staff. This applies particularly at primary level as this is a crucial time for the development of pupils' basic mathematical knowledge, skills and not least attitudes, and can be decisive in determining young people's response to the subject and whether they will engage in the field in the future. Initial teacher education programmes, assessment of prospective teachers, and CPD opportunities therefore need to refocus on promoting teachers' 'mathematical knowledge for teaching'. Furthermore, there must be an increased availability of specialist mathematics teachers, especially at primary level, if measurable improvements in student attainment are to be achieved.

The EACEA/Eurydice pilot survey of initial teacher education programmes (SITEP) revealed only minor differences between generalist and specialist teacher education programmes, although the response rates were low and therefore the data must be considered as an indication only. Both programmes placed high emphasis on the competences related to the teaching of the mathematics/science curriculum as well as on creating a rich spectrum of teaching situations. The differences identified were related to the higher level of importance given to the competences dealing with student diversity and gender issues in programmes for specialist teachers than for generalist teachers. However, the fact that these competences were least often addressed in both types of programmes indicates that there is a general need to strengthen teachers' knowledge and skills in these areas.

At present, centrally promoted programmes for professional development address a range of areas of competence that can encourage mathematics teachers to bring innovation to their mathematics teaching. However, international survey results show that low participation rates in such programmes, again especially among primary teachers, pose a problem that needs to be addressed. Topics that feature less in centrally promoted initiatives for professional development include gender sensitive mathematics teaching, the use of research findings, and the application of a range of assessment techniques. Yet, according to the findings of this report, these are precisely some of the key areas that need to be strengthened in mathematics teaching.

Finally, one area of professional development, namely teacher cooperation, collaboration, and exchange, is increasingly being promoted in most European countries, particularly in the form of online resources such as websites, blogs or other social networking sites. Research evidence confirms that it is crucial to support these online communities as they encourage teachers to learn from each other and can contribute to achieving progress on a larger scale.

G. Promoting evidence-based policies

Raising the quality of mathematics teaching also depends on the collection, analysis and dissemination of evidence on classroom practice and on what works in mathematics education. Moreover, common European objectives for reducing the numbers of students with inadequate mathematical skills and increasing the number of graduates in mathematics-related fields, necessarily requires strengthening the efforts to monitor and report on these areas at both national and European levels.

Research evidence and impact studies can inform policy development by indicating the extent to which new policies have been embedded in schools as well as highlighting practices that have proven to be successful. Some European countries report that information on classroom practice is being collected and analysed by teaching centres or research institutes that have been set up by education ministries themselves or by institutions which work in close collaboration with ministries. However, other countries have no such organisations to routinely carry out these types of activities.

Around half of European countries report investigating which teaching methods and activities are being used in mathematics lessons, while fewer countries look at the methods teachers use to assess their students. Clearly, this kind of information gathering can be expanded to both inform new policy decisions and evaluate the success of previous initiatives. Further national research could provide evidence about the effectiveness of specific approaches such as problem-based learning, real life contextualisation or the use of ICT, and highlight successful models which can be implemented in the classroom.

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GLOSSARY

Country codes

EU-27	European Union
BE	Belgium
BE fr	Belgium – French Community
BE de	Belgium – German-speaking Community
BE nl	Belgium – Flemish Community
BG	Bulgaria
CZ	Czech Republic
DK	Denmark
DE	Germany
EE	Estonia
IE	Ireland
EL	Greece
ES	Spain
FR	France
IT	Italy
CY	Cyprus
LV	Latvia
LT	Lithuania
LU	Luxembourg
HU	Hungary
MT	Malta
NL	The Netherlands

AT	Austria
PL	Poland
PT	Portugal
RO	Romania
SI	Slovenia
SK	Slovakia
FI	Finland
SE	Sweden
UK	The United Kingdom
UK-ENG	England
UK-WLS	Wales
UK-NIR	Northern Ireland
UK-SCT	Scotland
EFTA/EEA countries	The three countries of the European Free Trade Association which are members of the European Economic Area
IS	Iceland
LI	Liechtenstein
NO	Norway
Candidate country	
TR	Turkey

Statistical code

: Data not available

International Standard Classification of Education (ISCED 1997)

The international standard classification of education (ISCED) is an instrument suitable for compiling statistics on education internationally. It covers two cross-classification variables: levels and fields of education with the complementary dimensions of general/vocational/pre-vocational orientation and educational/labour market destination. The current version, ISCED 97 ([1]) distinguishes seven levels of education.

ISCED 97 LEVELS

Depending on the level and type of education concerned, there is a need to establish a hierarchical ranking system between main and subsidiary criteria (typical entrance qualification, minimum entrance requirement, minimum age, staff qualification, etc.).

ISCED 0: Pre-primary education

Pre-primary education is defined as the initial stage of organised instruction. It is school- or centre-based and is designed for children aged at least three years.

ISCED 1: Primary education

This level begins between four and seven years of age, is compulsory in all countries and generally lasts from five to six years.

ISCED 2: Lower secondary education

It continues the basic programmes of the primary level, although teaching is typically more subject-focused. Usually, the end of this level coincides with the end of compulsory education.

ISCED 3: Upper secondary education

This level generally begins at the end of compulsory education. The entrance age is typically 15 or 16 years. Entrance qualifications (end of compulsory education) and other minimum entry requirements are usually needed. Instruction is often more subject-oriented than at ISCED level 2. The typical duration of ISCED level 3 varies from two to five years.

ISCED 4: Post-secondary non-tertiary education

These programmes straddle the boundary between upper secondary and tertiary education. They serve to broaden the knowledge of ISCED level 3 graduates. Typical examples are programmes designed to prepare pupils for studies at level 5 or programmes designed to prepare pupils for direct labour market entry.

ISCED 5: Tertiary education (first stage)

Entry to these programmes normally requires the successful completion of ISCED level 3 or 4. This level includes tertiary programmes with academic orientation (type A) which are largely theoretically based and tertiary programmes with occupation orientation (type B) which are typically shorter than type A programmes and geared for entry into the labour market.

ISCED 6: Tertiary education (second stage)

This level is reserved for tertiary studies that lead to an advanced research qualification (Ph.D. or doctorate).

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(1) http://unescostat.unesco.org/en/pub/pub0.htm

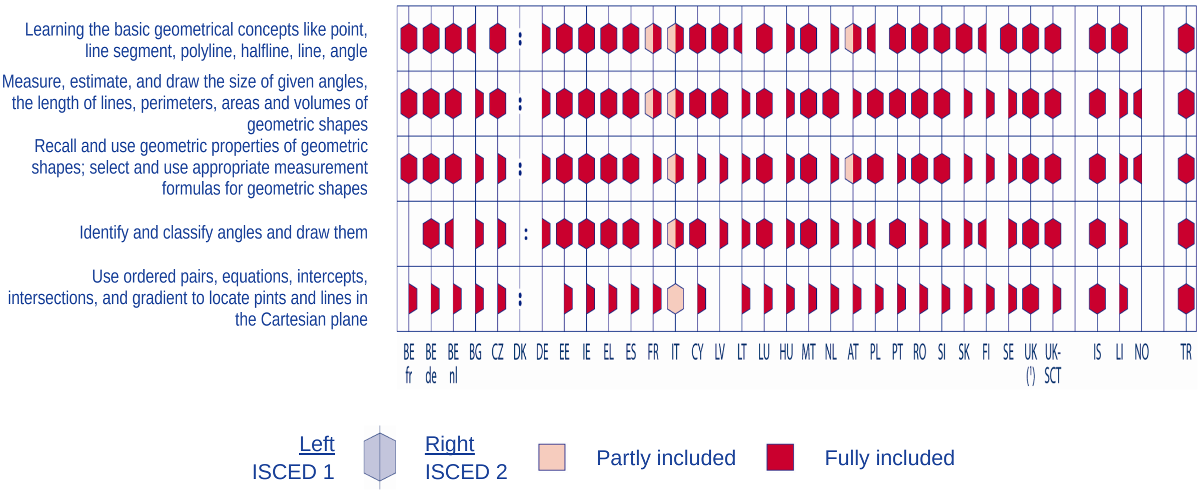
TABLE OF FIGURES

ANNEX 1 – Content of mathematics curriculum ([1]), 2010/11

1. Numbers

Source: Eurydice.

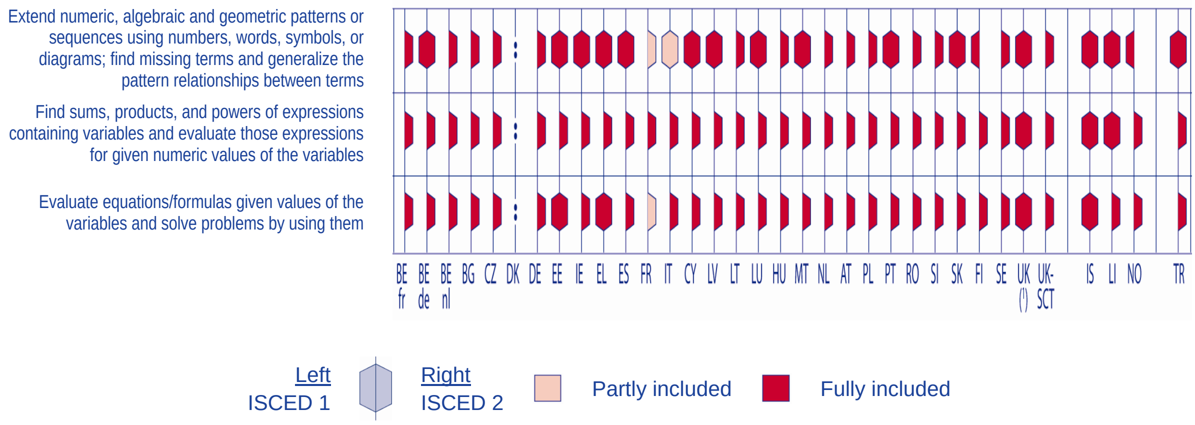
2. Geometry



Source: Eurydice.

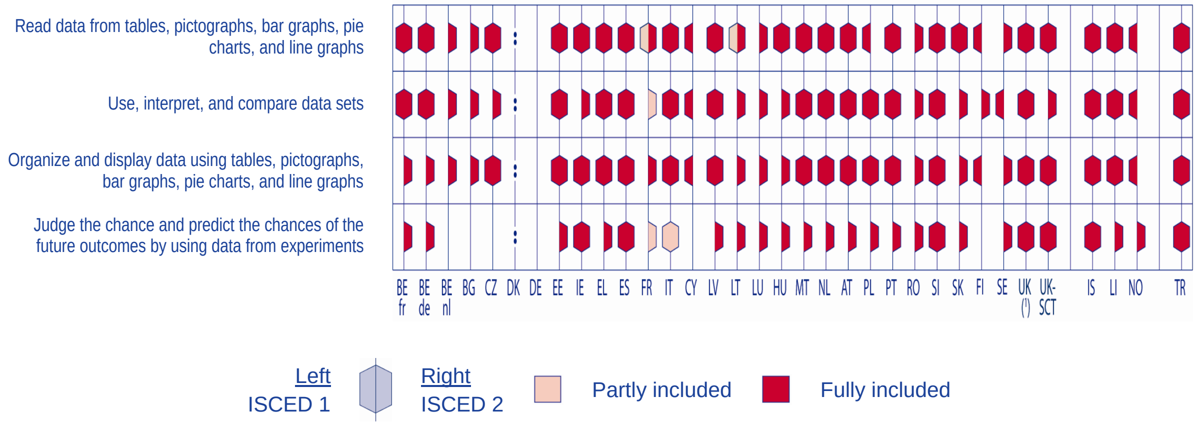
([1]) Based on the Mathematics knowledge areas used in TIMSS 2007 Curriculum Questionnaire. For more details, see Mullis et al., 2008.

3. Algebra



Source: Eurydice.

4. Data and chance



Source: Eurydice.

ANNEX 2 – Centrally promoted initiatives to encourage teacher collaboration, 2010/11

Belgium – French Community

- The official website for education organised by the French Community provides links to teaching resources placed online by teachers from compulsory education.
  http://www.restode.cfwb.be
- The official website of the French Community Education (www.enseignement.be) provides links to educational resources based on the subjects covered, including mathematics.
  http://www.enseignement.be/index.php?page=0&navi=184

Belgium – German-speaking Community

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Belgium – Flemish Community

- A general portal and share-site developed with the support of the Ministry of Education and Training, which includes an important mathematics section.

www.klascement.be

Bulgaria

- In partnership with Microsoft, a network of innovative teachers has been developed. Within the network, the registered users may share any learning content they create themselves; learn about good practices used by others; communicate with other members on issues related to the education system in general and about specific areas of interest; create blogs where they can create a personal profile and present their work, their involvement in projects, etc.

www.teacher.bg

- A popular network among teachers is the European network 'eTwining'. 'eTwinning' allows teachers from all around Europe to exchange information and experience in a safe virtual community. They implement common education projects, which are usually subject oriented and contribute to improving teaching methods and the atmosphere in the classroom.

http://www.etwinning.net/bg/pub/index.htm

Czech Republic

- The National Institute for Education, Education Counselling Centre and Centre for Continuing Education of Teachers (contributory organisation directly run by the Ministry of Education, Youth and Sports) is the responsible body for the 'Methodology Portal' it is operating. The aim of the portal is amongst others the improvement of the quality of the teaching profession through systematic support for teachers in teaching methodology and didactics; development of a learning community where teachers can share their experiences; use of effective methods of education in the lifelong learning of teachers.
- There is a wide range of material available on the portal arranged according to area of education, including mathematics. The portal offers articles, digital learning materials (worksheets, presentations, etc.), online community spaces (forums, wikis, 'digifolios', blogs) and e-learning courses. In addition to the electronic format, printed materials are also provided, such as article collections and Inspiromat (journal). Teachers' contributions to the website of examples of good practice are evaluated by a panel of experts.

http://rvp.cz/

Denmark

- The 'Educational Meeting Universe' provides teachers with a broad range of teaching resources for each subject, including mathematics. Teachers can also suggest teaching materials themselves.

www.emu.dk

Germany

- Teacher collaboration is promoted within the framework of the '*MINT Zukunft schaffen'* initiative. It is a nationwide, not-for-profit initiative, which was established in 2008 by German industry as a response to a skills shortage in professions related to mathematics, informatics, science and engineering. Part of the initiative is the MINT-Portal, a digital multiplier-platform that provides information about initiatives and projects that can be used by teachers to create interesting and attractive lessons.

http://www.mintzukunftschaffen.de

Estonia

- The project 'Raising the level of qualification for general education teachers 2008-2014' encourages the use of self-assessment methods by teachers and supports their knowledge of curriculum development, with a view to developing their professional skills and career opportunities. One of the project's goals is to create active cooperation environments (web-based or other) for the development and exchange of teaching and learning methods and materials.

http://www.ekk.edu.ee/programmid/programm-uldhariduse-opetajate-kvalifikatsioon

- The Estonian Mathematical Society and Community of School Mathematics Teachers organises a large variety of events for mathematics teachers, and is one of the principal bodies involved in generating and putting forward proposals for curriculum development.

  http://www.matemaatika.eu/

- The 'Mathematics Teachers' Day' is annual event where educators and teachers speak about the latest research results, ideas about good practice, etc. Speeches are published in reviewed set of articles entitled *Koolimatemaatika* (School Mathematics).

- Cooperation between mathematics teachers is also facilitated via the following networks:

  www.koolielu.ee
  http://mott.edu.ee/mottwiki/index.php/Esileht_(materials)
  http://www.geogebra.org/cms/et

- The project 'We love maths' *(Meile meeldib matemaatika)* includes a teachers' network that is supervised by teacher-educators of Tallinn University.

  http://zope.eenet.ee/mmmprojekt/

Ireland

- At primary level, a number of Teacher Professional Communities (TPC) relating to Maths Recovery have been established through the Teacher Education Network. Other TPCs relating to mathematics have also been established through the Teacher Education Network. The purpose of a TPC is to enable the collective development of new competences, new resources and new shared identities and motivation to work together for change.

  www.dwec.ie/programmes/tpc.html

- A number of websites also provide ideas and information/resource sharing opportunities for teachers. For example:

  http://ppds.ie/index.php?option=com_content&task=view&id=148&Itemid=459 ;
  http://www.ncte.ie/AdvancedSearch/?cx=011573740689929430170%3Ah0rwfmxhpfu&cof=FORID%3A11&ie=UTF-8&q=MATHEMATICS&siteurl=www.ncte.ie%2F#896 ; http://www.ncca.ie/en/Curriculum_and_Assessment/Assessment/

- The Irish Mathematics Teachers' Association supports the teaching of mathematics at all levels.

  www.imta.ie

Greece

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Spain

- On the website of the IFIIE (Institute for Teacher Training and Educational Research and Innovation), teachers can find sections linked to different issues; these provide training and teaching resources for teachers. For instance the CREADE (Resource Centre for the Attention to Cultural Diversity in Education) is an IFIIE project and, consequently, of the Ministry of Education. It was set up in response to professionals' interest in cultural diversity and its implications.

  http://www.boe.es/boe/dias/2006/12/08/pdfs/A43053-43102.pdf
  https://www.educacion.es/creade/index.do

- The INTERCAMBIA portal ('To educate in feminine and masculine') is a virtual space to share experiences about the interests, knowledge and motivations of girls, boys, men and women in education. It was created to facilitate access to and the exchange of information and knowledge about educational practices which include knowledge of gender issues. It resulted from an initiative of the Ministry of Education through the IFIIE, and the Ministry of Equality through the Institute of Woman, in collaboration with the equality bodies and the education authorities of the Autonomous Communities. The INTERCAMBIA portal is conceived as a 'virtual centre of thematic resources', a web space that gathers, recognises and disseminates the contributions to education of those men and women whose aim is to help educate in and for equal opportunities.

  https://www.educacion.es/intercambia/index.do

- The Institute for Educational Technologies includes among its aims: the development of the Ministry of Education's educational resources portal and the creation of social networks in order to facilitate the exchange of experiences and resources between teachers. It provides a digital network accessible to all teachers and makes available materials to which all teachers can contribute.

  http://www.ite.educacion.es/

- In each Autonomous Community, the Education Department supports Centres for Continuing Teacher Development with programmes to develop teacher networks. Examples of special websites developed by the Regional Education Departments include:

Andalusia:

http://www.juntadeandalucia.es/averroes/impe/web/portadaEntidad?pag=/contenidos/B/FormacionDelProfesorado/&textoPortada=no

Aragon:

http://www.educaragon.org/arboles/arbol.asp?guiaeducativa=42&strseccion=A1A31

Principality of Asturias:

http://www.educastur.es/index.php?option=com_content&task=category&sectionid=29&id=117&Itemid=124

Balearic Islands:

http://weib.caib.es/Formacio/contingut_for_.htm

Basque Country:

http://www.hezkuntza.ejgv.euskadi.net/r43-798/es/

Canary Islands:

http://www.gobiernodecanarias.org/educacion/pagina.asp?categoria=1523

Cantabria:

http://www.educantabria.es/formacion_del_profesorado/profesorado/formacionpermanente/modelodeformacion

Castile and Leon:

http://www.educa.jcyl.es/educacyl/cm/profesorado/tkContent?idContent=6991&locale=es_ES&textOnly=false

Castile-La Mancha:

http://www.educa.jccm.es/educa-jccm/cm/profesorado/tkContent?idContent=1641&locale=es_ES&textOnly=false

Catalonia:

http://www.xtec.net/formacio/index.htm

Community of Valencia:

http://www.edu.gva.es/per/es/sfp_0_sfp.asp

Extremadure:

http://www.educarex.es/

Galicia:

http://www.edu.xunta.es/web/taxonomy/term/63%2C153/all

Community of Madrid:

http://www.educa.madrid.org/educamadrid/

Region of Murcia:

http://www.carm.es/web/pagina?IDCONTENIDO=3918&IDTIPO=100&RASTRO=c908$m

Navarre:

http://www.educacion.navarra.es/portal/Formacion+del+Profesorado

La Rioja:

http://www.educarioja.org/educarioja/index.jsp?tab=prf&acc=crs&menu=2

France

- The website 'Eduscol', developed by the Ministry of Education, provides a variety of information related to school education for teaching professionals.

http://eduscol.education.fr/

Italy

- The 'GOLD' initiative promoted by ANSAS provides a website and database with a view to sharing, documenting and assessing good teaching practices.

http://gold.indire.it

Cyprus

- The Pedagogical Institute maintains an e-learning platform where teachers at all levels of education can find and share educational material as well as ideas.

http://www-elearn.pi.ac.cy/

Latvia

- A project team has created a network of 58 pilot and supporting schools. Seminars for observing and analysing lessons, sharing experience as well as for other activities are organised in these schools. Pilot schools organise similar activities independently for teachers of other schools not included in either the pilot or supporting school groups.

http://www.dzm.lv/par_projektu/skolas
http://www.dzm.lv/aktualitates/

Lithuania

- The project 'Networks of Cooperating Schools' aims to create the conditions for cooperating schools to improve the ability of their stakeholders, including teachers, to solve any problems that occur in relation to changes in the education process. The network also seeks to improve the quality of teaching and learning by helping to solve organisational problems; contributing to planning the content of education, dealing with students' lack of motivation to learn, meeting students' needs, etc.

http://www.bmt.smm.lt/?age_id=8

Luxembourg

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Hungary

- The 'Bolyai Mathematical Society' is regarded by the government as an official teachers' network. It is itself a member of the International Mathematical Union and the European Mathematical Society. The Ministry of National Resources consults the Society over every education policy issues relating to mathematics. The Society has approximately 600-700 mathematics teachers as members. Its goals include promoting research in mathematics; promoting mathematics and its wider use; solving issues relating to the teaching of mathematics; representing the interests of mathematics professionals and supplying information about researchers, experts and teachers. To achieve these goals, the Society creates opportunities to publish and discuss new results, education policy and scientific issues in mathematics; and organises in-service training for teachers, camps for students, conferences, and seminars independently or together with other organisations.

http://www.bolyai.hu/

Malta

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The Netherlands

- The Ministry of Education has supported a mathematics teacher training network (ELWIER) for about 5 years. The network allows teachers to meet and develop teaching materials for mathematics.

www.elwier.nl

- Panama is a networking project for everyone involved in the field of arithmetic-mathematics in primary education including primary teacher training and training for teaching assistants. The focus is on educators, counsellors and researchers. Panama offers a platform for exchanging expertise, experiences and ideas. The activities organised by Panama aim to contribute to the development of good mathematics education, this includes the implementation of new knowledge and developments in primary level mathematics education.

www.fi.uu.nl/panama

Austria

- IMST Regional Networks: These regional programmes launched by IMST *(Innovationen Machen Schulen Top)* often work in mathematics as well as other science subjects. The main aims are to improve the quality of teaching and make it more appealing to students, to develop the skills and professionalism of teachers, and to include as many schools and school types as possible within the networks. Each regional network works on the basis of a contract between IMST and the corresponding school-board of the federal state and each one has a steering group.

  http://imst.uni-klu.ac.at/programme_prinzipien/rn_tn/

- In each province of Austria, there are working groups *(Arbeitsgemeinschaften)* for mathematics which organise meetings of mathematics teachers often focusing on a certain topic, such as the new nationally standardised upper secondary leaving examination *(Zentralmatura)*.

  Steiermark: http://arge.stvg.at/arge.nsf

  Salzburg: http://schule.salzburg.at//faecher/mathematik/minhalt.htm

- 'proMath' is an initiative of the Ministry for Education, the Arts and Culture that offers online-services for the teaching and learning of mathematics to teachers, students, and parents at middle-level and higher-level technical and vocational schools.

  http://www.promath.tsn.at/

Poland

- The 'Scholaris-Web Centre of Educational Resources' is an initiative of the Ministry of National Education which provides an online space where teachers can exchange teaching materials and resources.

  http://www.scholaris.pl/

Portugal

- One of the goals of the 'Teacher Education Programme in Mathematics' is to provide each school cluster with specialists in mathematics teaching as well as to create and disseminate national materials for teaching mathematics. The results show that these goals have been reached: teachers organise seminars to discuss their practices, experiences and activities; teachers share documents, lessons plans and tasks, and discuss them; the programme has also strengthened collaboration between teachers and researchers.

  http://www.dgidc.min-edu.pt/outrosprojetos/index.php?s=directorio&pid=31

Romania

- The Romanian Mathematical Society publishes two Mathematics Journals: type A – for teachers and type B – for students. It also organises contests, conferences, and educational projects.

  http://rms.unibuc.ro/

- The Institute of Mathematics 'Simion Stoilow' (IMAR) is one of the research institutes of the Romanian Academy, representing one of the most significant centres of Romanian mathematical activity. During its 50 years of operation virtually all leading Romanian mathematicians have been members or associated somehow with this research organisation.

  http://www.imar.ro/

Slovenia

- Teachers' networks exist and are included in many projects, programmes and professional development seminars for teachers. Of special interest is the virtual classroom run by the National Education Institute which provides a very good link between teachers and experts in specialist mathematics didactics.

  http://skupnost.sio.si/mod/wiki/view.php?id=73919&page=Matematika

Slovakia

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Finland

- The LUMA Centre is an umbrella organisation for the cooperation of schools, universities, business and industry which is coordinated by the Faculty of Science of the University of Helsinki. The goal is to support and promote the teaching and learning of science, mathematics and technology, at all levels. One of the main aims of the LUMA Centre is to support teachers' lifelong learning. Workshops, summer courses and an annual LUMA Science Fair are organised for subject teachers and primary school teachers. Information on coming events, new teaching materials and research findings is available in a monthly email newsletter and on the webzine LUMA Sanomat. In addition to this, the resource centres support subject-specific activities with the material available on their websites. Question and discussion forums are another form of activity organised by the centre. Finally, disseminating new research findings is key in supporting teachers' lifelong learning. This is done with the help of LUMA Science Fairs and summer courses, and by offering the opportunity to take part in research and to follow new developments through the newsletter, the webzine Luova and Master's theses published by the resource centres. 'The Researcher of the Month' column is published in the LUMA Newsletter and in the webzine Luova.

  http://www.helsinki.fi/luma/luma2/english/

Sweden

- The National Centre for Mathematics Education (NCM), run by the University of Gothenburg, is the Swedish national resource centre for mathematics. Its main task is to support the development of Swedish mathematics education in pre-school, school and adult education. Activities include conferences, courses, workshops, research and development, a national reference library, teaching material, advisory service and development support.

  http://ncm.gu.se/english

- The Swedish National Agency for Education website *(Skolverket webbplats)* gathers a lot of materials for teachers' use, allows the exchange of information, provides newsletters, etc.

  http://www.skolverket.se/

The United Kingdom – England

- The National Centre for Excellence in the Teaching of Mathematics (NCETM) aims to meet the professional aspirations and needs of all teachers of mathematics and realise the potential of learners through a sustainable national infrastructure for mathematics-specific continuing professional development (CPD).

  The NCETM provides and signposts high quality resources to teachers, mathematics education networks, HEIs and CPD providers throughout England. At the same time, the National Centre encourages schools and colleges to learn from their own best practice through collaboration among staff and by sharing good practice locally, regionally and nationally.

  This collaboration takes place virtually through the NCETM portal and 'face to face' through a network of regional coordinators in nine regions throughout England. The portal is becoming one of the main places on the web where mathematics teachers can go to find reliable information about teaching methods, resources, research findings and CPD opportunities. The regional coordinators raise awareness of regional and national CPD opportunities, establish links with the regional professional development infrastructure and facilitate meetings, activities and collaborative projects.

  The Centre also funds and publishes research into effective mathematics teaching practices and CPD. Teacher enquiry in the classroom is supported by the Centre and findings are shared via the portal. Research both informs the NCETM's strategy and acts as a form of CPD in its own right.

  https://www.ncetm.org.uk

The United Kingdom – Wales

- A National Science Academy (NSA) has been established in Wales to promote the take up of science, technology, engineering and mathematics – STEM subjects – at all levels to ensure Wales has a continuous supply of people graduating from colleges and universities with the appropriate qualifications and skills.

  http://wales.gov.uk/topics/educationandskills/allsectorpolicies/nsa/?lang=en

The United Kingdom – Northern Ireland

- In Northern Ireland, the government has funded the STEM module which is a mobile laboratory and workshop designed to bring high quality learning experiences in STEM subjects to pupils in 17 STEM specialist schools and associated learning communities.

  http://www.education-support.org.uk/stem

The United Kingdom – Scotland

- A major support for teachers is Glow. Glow is the world's first national intranet for education that is transforming the way the curriculum is delivered in Scotland. Every teacher in Scotland has access to Glow and can use the facility to communicate with any other teacher in Scotland through a range of open forum facilities or through video conferencing. The system also allows any teacher to upload work, ideas or other documents that can then be shared nationally.

  There is a national Glow group for mathematics and another for numeracy. The facility also allows access to all of Scotland's pupils. Levels of restriction can be part of the group access and this allows for the appropriate degree of discretion. The national Glow groups for mathematics and numeracy also contain notes on forthcoming events, national and international developments and links to web sites that have been highlighted as useful. The reason for the stand alone numeracy group is that all teachers in Scotland have a responsibility for the development of this subset of mathematics and it was felt that non-maths specialists would be more inclined to interact with a numeracy site than a site with the more abstract areas of mathematics.

  http://www.ltscotland.org.uk/usingglowandict/index.asp

- Another mathematics network that is supported at central level is the 'Mathematics Advisory Group for Scotland' (MAGS). MAGS meet four times a year and representatives from all EAs are invited to attend. The meetings share national and international developments, get feedback from individual EAs on work in hand and invite key partners (Her Majesty's Inspectorate of Education, Learning and Teaching Scotland (HMIE) and the Scottish Qualifications Authority) to provide updates on national matters. MAGS caters for primary and secondary teachers and tries to get classroom practitioners to share experiences.

- The Scottish Mathematical Council (SMC) is another key network body for the development of mathematics. The SMC is more focussed on secondary education and has representation from universities. The major CPD opportunity for mathematics teachers in Scotland is the annual SMC Conference. It is held in early March and attracts about five hundred delegates who can choose from up to thirty workshops. The workshops are provided by classroom practitioners, HMIE, SQA, LTS and high profile national and international researchers.

  http://scottishmathematicalcouncil.org/index.php?option=com_content&task=view&id=3&Itemid=1

Iceland

- The Association of Mathematics Teachers is supported by The Ministry of Education. A grant was awarded to the association for the school year 2010/11 to support meetings and conferences on educational issues that can improve professional development (not individual grants). It is also used to promote curriculum materials, teaching methods, assessment and other related issues through newsletters and websites.

  http://flotur.ismennt.is

Norway

- Mathematics development is fostered through the National Centre for Mathematics in Education. Its main objective is to lead and coordinate the development of new and better teaching methods and tools for mathematics education in kindergarten, primary and secondary schools, adult education and teacher training. The Centre actively promotes innovation, debate and the sharing of experience within the discipline. The Centre's target audience are teachers of mathematics in schools and teacher training, teachers and students at colleges and universities, and developers of learning materials. To build up a positive view of mathematics in society in general, parents, the media and the public are also important targets for the Centre's activities.

  http://www.matematikksenteret.no

- The National Directorate for Education and Training website offers resources for teaching, guidelines for schools, etc. related to different methods of teaching in mathematical topics.

  http://www.udir.no/

- *Skole i praksis* (School in practice) offers a series of film-based resources for mathematics instruction.

  http://www.skoleipraksis.no/

Turkey

- The Ministry of National Education's website is the main portal for all information concerning school education.

  http://www.meb.gov.tr/

ANNEX 3 – Response rates by country from the Survey on Initial Teacher Education Programmes in Mathematics and Science (SITEP)

	Available programmes	Institutions	Replies by programme	Replies by institution	Rate of response by programmes	Rate of response by institutions
Belgium (French Community)	39	16	2	2	5.13	12.50
Belgium (German-speaking Community)	:	:	NA	NA	NA	NA
Belgium (Flemish Community)	31	18	13	9	41.94	50.00
Bulgaria	33	8	2	2	6.06	25.00
Czech Republic	80	12	25	12	31.25	100.00
Denmark	14	7	6	6	42.86	85.71
Germany	469	144	41	32	8.74	22.22
Estonia	11	2	2	1	18.18	50.00
Ireland	23	20	2	2	8.70	10.00
Greece	33	9	4	4	12.12	44.44
Spain	110	51	26	16	23.64	31.37
France	91	33	4	4	4.40	12.12
Italy	24	24	4	3	16.67	12.50
Cyprus	5	4	0	0	0.00	0.00
Latvia	19	5	7	5	36.84	100.00
Lithuania	24	8	3	1	12.50	12.50
Luxembourg	2	1	2	1	100.00	100.00
Hungary	38	17	8	7	21.05	41.18
Malta	2	1	2	1	100.00	100.00
Netherlands	96	45	10	8	10.42	17.78
Austria	35	18	14	8	40.00	44.44
Poland	163	95	12	8	7.36	8.42
Portugal	93	42	8	8	8.60	19.05
Romania	80	27	5	4	6.25	14.81
Slovenia	29	3	1	1	3.45	33.33
Slovakia	24	11	3	2	12.50	18.18
Finland	14	8	2	2	14.29	25.00
Sweden	55	22	1	1	1.82	4.55
United Kingdom (England)	347	70	45	33	12.97	47.14
United Kingdom (Wales)	21	6	4	4	19.05	66.67
United Kingdom (Northern Ireland)	12	4	3	1	25.00	25.00
United Kingdom (Scotland)	35	8	7	6	20.00	75.00
Iceland	2	2	0	0	0.00	0.00
Liechtenstein	:	:	NA	NA	NA	NA
Norway	16	16	1	1	6.25	6.25
Turkey	155	58	13	10	8.39	17.24
TOTAL	2 225	815	282	205		

EDUCATION, AUDIOVISUAL AND CULTURE EXECUTIVE AGENCY

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EN

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